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THE DESIGN AND THE OPTIMIZATION OF THE FORK-PIN COMPRESSION JOINTS IN FRONT MOTORBIKE SUSPENSIONS

D. Croccolo, R. Cuppini and N. Vincenzi

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1. Introduction

The design of the fork-pin compression-fit joints of front motorbike suspensions is uncertain mainly because of the poor knowledge about the starting friction coefficient μ_{II} and about the mean coupling pressure p. The axial releasing force $F_{II}=\mu_{II}\cdot p\cdot A$, which is the fundamental design parameter, depends on the mentioned two factors, usually unknown, and on the coupling surface dimension, usually known. For these reasons, we decided to develop a generalized methodology which may be able to calculate the parameters (μ_{II} and p) of fork-pin joints like those reported in Figure 1 and produced by Paioli Meccanica S.p.A.. In a previous work [Croccolo, 2002], the mathematical model useful to calculate the starting friction coefficient μ_{II} has been defined as a function of the production and assembly specifications which are the resting time, the presence of humidity, and the presence of protecting oil. Instead, the definition of the coupling pressure is more difficult because it requires the help of numerical (FEM) analyses. In fact, cause to the not axial symmetric geometry of the fork, the tensile state on the coupling surfaces does not result constant as shown by the different contours of the images of the Figure 2 and therefore it is not possible to apply the high thickness pipe theory. Furthermore, the FEM analysis needs to use many contact elements which increase the computational time and which often do not provide any solution. On the other hand, the pin has, normally, a perfect axial symmetric geometry which is simple to study with the theoretical formulas. Therefore, we decided to separate the strain contribution of the pin and of the fork going on improving the methodology presented in [Croccolo, 2002] and [Croccolo, 2003] which allows to continue in applying the theoretical formulas appropriately corrected. The mathematical models of μ_{II} and p are, therefore, implemented in a program which is useful to quickly perform the design and the verification of the joint without using the FEM analyses.

Figure 1. An example of a fork-pin joint

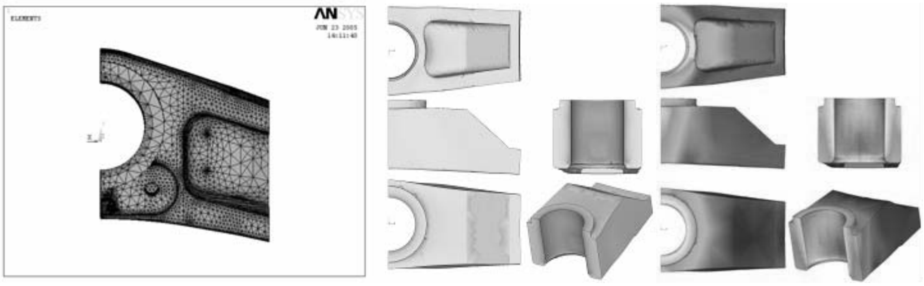

Figure 2. FEM models and σ_r and σ_t distributions for an imposed Z displacement

2. Legend of symbol

μ_{ll}	starting friction coefficient	ε_t	tangential strain
F_{ll}	axial releasing force	E_A	fork Young modulus
De_t, Di_t	external, internal diameter (fork)	ν_A	fork Poisson coefficient
De_p, Di_p	external, internal diameter (pin)	E_I	pin Young modulus
D_F	nominal joint diameter	ν_I	pin Poisson coefficient
L_acc	joint length	p_{acc_r}	real mean pressure
L_s	hollow dimension	p_{acc_th}	theoretical mean pressure
Q_A=Di_t/De_t	diameters ratio (fork)	σ_{r_r}	real radial tension
Q_I=Di_p/De_p	diameters ratio (pin)	σ_{r_th}	theoretical radial tension
s	fork collar thickness	σ_{t_r}	real tangential tension
j	transversal stiffness parameter	σ_{t_th}	theoretical tangential tension
k	longitudinal stiffness parameter	$\beta_r=\sigma_{r\_r}\backslash\sigma_{r\_th}$	radial stresses ratio
Z_{AR}	fork radial interference	$\beta_t=\sigma_{t\_r}\backslash\sigma_{t\_th}$	tangential stresses ratio
Z_{IR}	pin radial interference		
Z_{totR}	total radial interference		

3. Methodology

Our investigations were basically dedicated to find out an overall mathematical function β which depends on some geometric parameters of the fork and which is able to correct the theoretical formulas valid for axial-symmetric elements. Hence, it is possible to design the joint and to compare some different solutions without performing the FEM analyses.

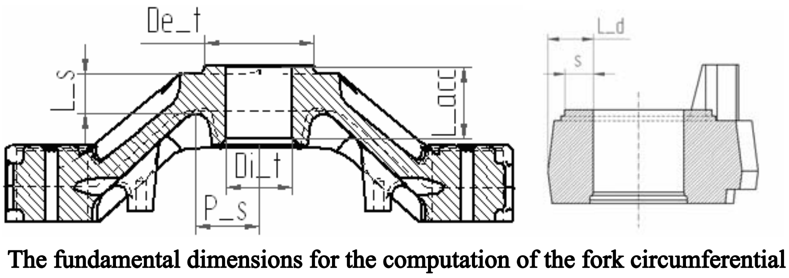

Figure 3. The fundamental dimensions for the computation of the fork circumferential stiffness

Unfortunately, the theoretical formulas do not provide accurate results (in some case the errors are higher than 60%) because of the different circumferential stiffness of the fork which seems to be influenced by the geometrical parameters located near the coupling zone (Figure 3).

First of all, we analysed the trend of the mean contact pressure, both theoretical (p_{acc_th}) and real (p_{acc_r}), calculated for different combinations of the internal joint diameter Di_t and of the collar thickness s of the fork. The theoretical values have been calculated applying (1) and considering the forks as cylinders with internal diameter Di_t and external diameter De_t=Di_t+2s (in (1) the subscript

($_A$) refers to the fork). The values of the real contact pressure, instead, have been determined by performing some FEM analyses on simplified fork models (Figure 2) with different Di_t and s combinations. We considered six different diameters Di_t (25mm, 27mm, 29mm, 31mm, 33mm, 35mm) and five different thicknesses s (6mm, 7,3mm, 8mm, 8,5mm, 9mm) for a total of 30 analysis for each group of different forks. The fork constrains, referring to the Figure 2, are:

- the nodes on the intersection between the plane of symmetry (Y, Z) and the fork have no translations in X axis direction and no rotation around Y and Z axes;
- the nodes on the two upper segments in this section (parallel to the Y axis) have no translations in Z axis direction.

For every fork of each group, we imposed a constant displacement (Z_{AR}) on the whole coupling surface in the circumferential direction, and we computed the p_{acc_r} and σ_{t_r} as the averages of the values of the pressure and of the tangential tension on the contact surface nodes. It was noticed that the trend of the theoretical and real pressure is the same if they are plotted as a function of the Di_t and the s.

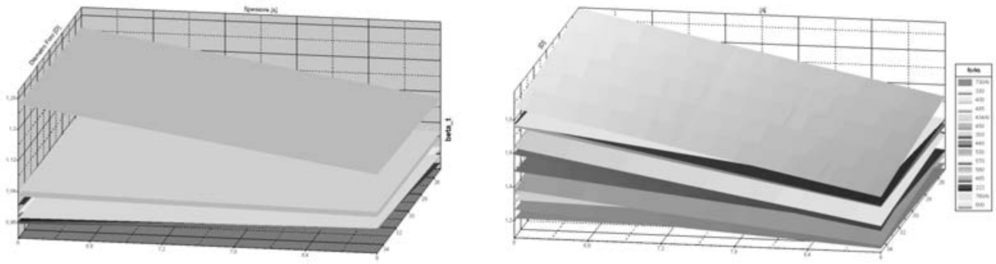

Figure 4. β_t and β_r diagrams depending on Di_t and s (the equations are reported in Table 1)

Then, each value of the mean pressure obtained by the FEM analyses have been compared with those obtained by applying the theoretical formulas. We therefore calculated the β_r (2) and the β_t (3) coefficients as ratio between the real tension (radial and tangential in average) and the theoretical one. For each different fork, β coefficients depend on the internal diameter Di_t and on the collar thickness s whereas are independent from the displacements Z_{AR}. Furthermore, as β ratios trends are similar to planes (Figure 4), they were interpolated obtaining some linear functions which depend on Di_t and s parameters [Croccolo, 2002], [Croccolo, 2003] and may be used for similar shape of forks in (11), (12) and (13).

The final step was to define an overall mathematical function able to interpolate all the β coefficients calculated for the different forks. The overall function for β_t coefficients was easily found choosing the β_t mean plane function (4) which introduces errors lower than 4% for all the forks. Unfortunately the differences and, therefore, the errors between the β_r planes are too high and, therefore, it was impossible to choose the β_r mean plane function . Hence, it was necessary to define and to apply other two parameters, j (5) and k (6), which are able to point out the fork stiffness across the two orthogonal directions, transversal and longitudinal, of the fork. The overall expressions for β_t (4) and for β_r (7) can, therefore, be used for the whole Paioli's fork production, both in steel and in aluminium.

$$p_{acc\_th} = \frac{2 \cdot Z_{AR} \cdot E_A}{Di\_t \cdot \left(\frac{1+Q_A^2}{1-Q_A^2} + \nu_A \right)} \tag{1}$$

$$\beta_r = \frac{p_{acc\_r}}{p_{acc\_th}} = \frac{\sigma_{r\_r}}{\sigma_{r\_th}} \tag{2}$$

$$\beta_t = \frac{\sigma_{t\_r}}{\sigma_{t\_th}} \tag{3}$$

$$\beta_t = 0{,}831 + 0{,}003 \cdot Di\_t + 0{,}007 \cdot s \tag{4}$$

$$j = \frac{L\_s}{L\_acc} \cdot \left(P\_s - \frac{Di\_t}{2} \right) \tag{5}$$

$$k = \frac{s}{L\_d} \tag{6}$$

$$\beta_r = 2{,}0008 + 0{,}0022 \cdot Di\_t - 0{,}0714 \cdot s + 0{,}0372 \cdot j - 0{,}4597 \cdot k \tag{7}$$

3.1 The mathematical model

The expressions of the displacements Z in the shaft-hub axial symmetrical joints are the well known following formulas in which the subscript ($_I$) refers to the shaft and the subscript ($_A$) refers to the hub:

$$\begin{cases} Z_{IR} = -\dfrac{\varepsilon_{t\_Ie} \cdot De\_p}{2} = -\dfrac{De\_p}{2 \cdot E_I} \cdot \left(\sigma_{t\_th\_Ie} - \nu_I \cdot \sigma_{r\_th\_Ie} \right) = -\dfrac{De\_p}{2 \cdot E_I} \cdot p_{acc\_th} \cdot \left(\dfrac{1+Q_I^2}{1-Q_I^2} - \nu_I \right) \\ Z_{AR} = \dfrac{\varepsilon_{t\_Ai} \cdot Di\_t}{2} = \dfrac{Di\_t}{2 \cdot E_A} \cdot \left(\sigma_{t\_th\_Ai} - \nu_A \cdot \sigma_{r\_th\_Ai} \right) = \dfrac{Di\_t}{2 \cdot E_A} \cdot p_{acc\_th} \cdot \left(\dfrac{1+Q_A^2}{1-Q_A^2} + \nu_A \right) \end{cases} \tag{8}$$

$$Z_{totR} = Z_{AR} - Z_{IR} = p_{acc\_th} \cdot \left(\frac{Di\_t}{2 \cdot E_A} \cdot \left(\frac{1+Q_A^2}{1-Q_A^2} + \nu_A \right) + \frac{De\_p}{2 \cdot E_I} \cdot \left(\frac{1+Q_I^2}{1-Q_I^2} - \nu_I \right) \right) \tag{9}$$

The (8) and the (9) do not provide any error if the geometry is cylindrical or axial-symmetric because the strain ε_t and the pressure p are constant across the whole contact surface.
In the case of the fork-pin joints the (8) and the (9) lead to wrong results because of the asymmetry of the fork, which is far from the cylindrical shape. The main causes of the errors are:
the not correct definition of the parameter Q_A=Di_t/De_t due to the variability of the external diameter of the fork as shown in Figure 3 (approximately De_t=Di_t+2s);
the not constant value of the pressure p across the whole coupling surface due to the different circumferential stiffness of the fork.
Hence, the real value of the coupling pressure may be calculated by the FEM analysis which has to be performed for each different type of joint. Obviously, this procedure is not fast nor easy to perform. Conversely, it is possible to try to correct the (8) and the (9) introducing the β coefficients in order to evaluate the real mean tensile state. The theoretical formulas of congruence and equilibrium (8) and (9) may be, therefore, rewritten in (10) and (11) where the β_r(Di_t,s,j,k) coefficient is used in order to correct the (8) and the (9) and to calculate the real value of the mean coupling pressure, p_{acc_r} (12), without the FEM analysis application. Finally the β_t(Di_t,s) is useful to calculate the real value of the mean coupling tangential stress, σ_t (13).

$$\begin{cases} Z_{IR} = -\dfrac{De\_p}{2 \cdot E_I} \cdot p_{acc\_r} \cdot \left(\dfrac{1+Q_I^2}{1-Q_I^2} - \nu_I \right) \\ Z_{AR} = \dfrac{Di\_t}{2 \cdot E_A \cdot \beta_r(Di\_t,s,j,k)} \cdot p_{acc\_r} \cdot \left(\dfrac{1+Q_A^2}{1-Q_A^2} + \nu_A \right) \end{cases} \tag{10}$$

$$Z_{totR} = Z_{AR} - Z_{IR} = p_{acc\_r} \cdot \left(\frac{Di\_t}{2 \cdot E_A \cdot \beta_r(Di\_t,s,j,k)} \cdot \left(\frac{1+Q_A^2}{1-Q_A^2} + \nu_A \right) + \frac{De\_p}{2 \cdot E_I} \cdot \left(\frac{1+Q_I^2}{1-Q_I^2} - \nu_I \right) \right) \tag{11}$$

$$p_{acc\_r} = Z_{AR} - Z_{IR} = \frac{Z_{totR}}{\dfrac{D_F}{2} \left(\dfrac{1}{E_A \cdot \beta_r(Di\_t,s,j,k)} \cdot \left(\dfrac{1+Q_A^2}{1-Q_A^2} + \nu_A \right) + \dfrac{1}{E_I} \cdot \left(\dfrac{1+Q_I^2}{1-Q_I^2} - \nu_I \right) \right)} \tag{12}$$

$$\sigma_{tAi\_reale} = \sigma_{tAi\_teorica} \cdot \beta_t(Di\_t,s) = p_{acc\_t} \cdot \beta_t(Di\_t,s) \cdot \frac{1+Q_A^2}{1-Q_A^2} = p_{acc\_r} \cdot \frac{\beta_t(Di\_t,s)}{\beta_r(Di\_t,s,j,k)} \cdot \frac{1+Q_A^2}{1-Q_A^2} \tag{13}$$

Once the pressure p_{acc_r}, the coupling surface A, and the starting friction coefficient μ_{II} are known, it is possible to calculate the releasing axial force of the joint using the formula $F_{II}=\mu_{II} \cdot p_{acc\_r} \cdot A$.

4. Results

We had analyzed a total of fifteen forks, made both in steel and in aluminium. For each fork we defined a group of 30 elements on which we had performed about 500 FEM analyses obtaining all the planes shown in Figure 4 and described by the equations reported in Table 1.

Table 1. β_r and β_t equations for the 15 analyzed forks

Fork identification number	β_r(Di_t,s)	β_t(Di_t,s)
949.115.222	2,647-0,002D-0,134s	0,802+0,003D+0,008s
949.115.440	2,640-0,009D-0,122s	0,789+0,004D+0,008s
949.115.465	2,493-0,001D-0,129s	0,855+0,002D+0,007s
P366_303	1,800-0,005D-0,078s	0,938+0,001D+0,004s
330	1,953-0,005D-0,068s	0,868+0,002D+0,004s
949.115.570	2,468-0,005D-0,108s	0,734+0,006D+0,011s
949.115.530	2,732-0,001D-0,125s	0,777+0,005D+0,009s
949.115.600	3,102-0,010D-0,146s	0,798+0,006D+0,010s
949.115.400	1,503-0,004D-0,046s	0,989+0,001D+0,003s
949.115.560	2,551-0,005D-0,124s	0,808+0,004D+0,008s
949.115.450	2,090-0,007D-0,803s	0,889+0,003D+0,005s
949.115.485	1,492+0,0018D-0,048s	0,977+0,002D+0,001s
949.113.434	1,174+0,074D-0,018s	1,028+0,001D-0,005s
949.113.730	1,067+0,006D-0,008s	1,044+0,001D-0,055s
949.113.760	2,016+0,009D-0,093s	0,861+0,003D+0,003s

The numerical analyses pointed out that the tangential tensions are very similar for the fork of each group and that they are very close to their theoretical values, while the radial tension are very different from the theoretical ones (in some fork the difference is higher than 60%) and also inside each group the discrepancy is quite significant. The p_{acc_r} values (real) are always greater than the p_{acc_th} values (theoretical) and they have the same trend when the collar thickness and the joint diameter change because the fork geometry is stiffer than the cylindrical one corresponding to the collar dimension: hence, the stiffest forks have the highest β_r. Anyway, the β coefficient trends, analyzed inside each group, are regular and almost similar to planes (Figure 4); furthermore, they are independent from Z displacements [Croccolo, 2002]. Therefore, it is possible to interpolate the β values, evaluated for each fork type, with equations of planes which only depend on Di_t and s (Table 1). These equations produce errors always lower than 4% for each group. Moreover, all the planes of β_t(Di_t, s) are very close each other and therefore they were interpolated with a mean equation (4) which provides errors always lower than 5%. Instead, the planes of β_r(Di_t, s) are quite different, with a discrepancy up to 30%, because the difference of the forks stiffness is not appreciable with the two parameters Di_t and s.

Carefully analyzing the forks geometries, we noticed that the stiffness is influenced both in the transversal direction and in the longitudinal direction by the density of the material located near the central bush (Figure 4). We, therefore, defined two additional parameters which were introduced in the analytical expression of β_r in order to interpolate all the planes with the same equation. These additional parameters, j (5) and k (6), are able to estimate the density of the material around the central bush and, at the same time, they are easy to be pointed out in all the forks. The j parameter, as shown in Figure 4, depends on:

- L_s: the height of the hollow, evaluated from the top, which may be located near the collar (proportional to the stiffness);
- P_s: the position of the apex of the above mentioned hollow referring to the central hole (proportional to the stiffness);
- L_acc: the length of the joint (proportional to the stiffness).

The k parameter instead depends on:

- s: the thickness of the collar;
- L_d: the maximum width of the front part of the fork in the plant view.

According to the previous definitions, the j parameter is proportional to the fork stiffness across the transversal direction and therefore the stiffest forks have the j close to one; the k parameter is proportional to the fork compliance in the longitudinal direction, and therefore when the k is close to one the fork material does not exceed the central bush. The β_r equation (7) is hence capable to interpolate all the analyzed forks with errors lower than 10%.

The j parameter has been evaluated through a genetic algorithm which was able to estimate the β_r coefficients which introduce the lowest errors compared with the FEM analyses. In other words, the algorithm combines the geometrical quantities L_s, P_s, and L_acc in many different ways until it finds the combination that minimizes the error. We considered a good solution when the error is lower than 10%. The errors evaluation was done through a program realized in Maple R8® environment which was useful to define the j parameter (5); conversely, the k parameter (6) was defined analysing carefully the fork drawings. Finally, we introduced all the defined parameters (Di_t, s, j and k) in an appropriate polynomial interpolator (7); in those equation if L_s is equal to L_acc j has to be set equal to s/2.

The equation (7) is able to interpolate every type of fork produced by the Paioli Meccanica S.p.A. with errors lower than 10%. The errors change in function of the collar thickness s and of the fork internal diameter Di_t. We noticed that in order to keep the error low, the thickness of the collar should be at least 8mm and the internal diameter at least 31mm. Fortunately, these features refer to the 99% of Paioli's production; in this case the errors are always lower than 10%. Instead, the errors produced by the βt(Di_t, s) (4) are always lower than 5%, and this because the βt planes are very close each other which is the reason because the βt coefficients are defined only in function of Di_t and s.

Finally, we realized an original program which executes, in a guided way, the design and the verification of the fork-pin joints. The input windows (Figures 5 and 6) show the geometrical characteristics and the information about the materials, the surface finishing and the parameters of the production and the assembly conditions. The output windows (Figure 7) shows all the project parameters calculated and provided by the program; in particular the program provides the maximum and the minimum value of the interference, the maximum and the minimum value of the axial releasing force and the fit standardized joint. Then, the user can accept the proposed joint or modify it importing the output project data of Figure 7 into the input window of the verification section of the program. Finally, the output window of the verification section (Figure 7) shows the value of the minimum and the maximum interference, the minimum and the maximum axial releasing force and the safety coefficient regarding to the yield point of the material. The calculus can be carried out applying the proposed corrected formulas or the theoretical ones.

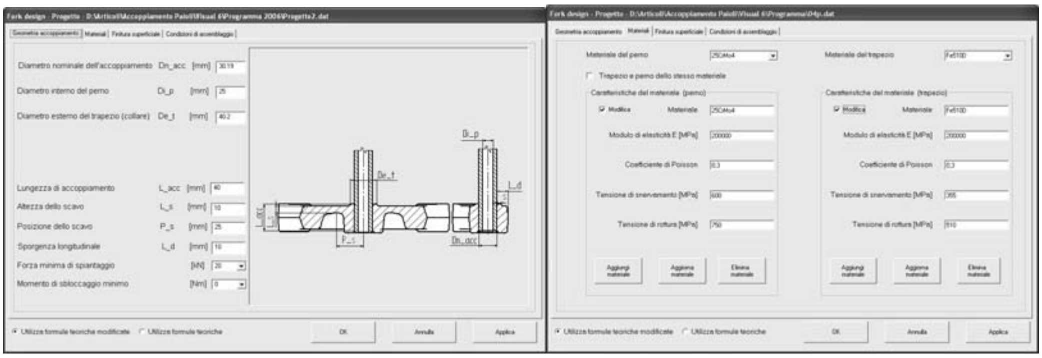

Figure 5. Input windows of the Fork Design program (design phase)

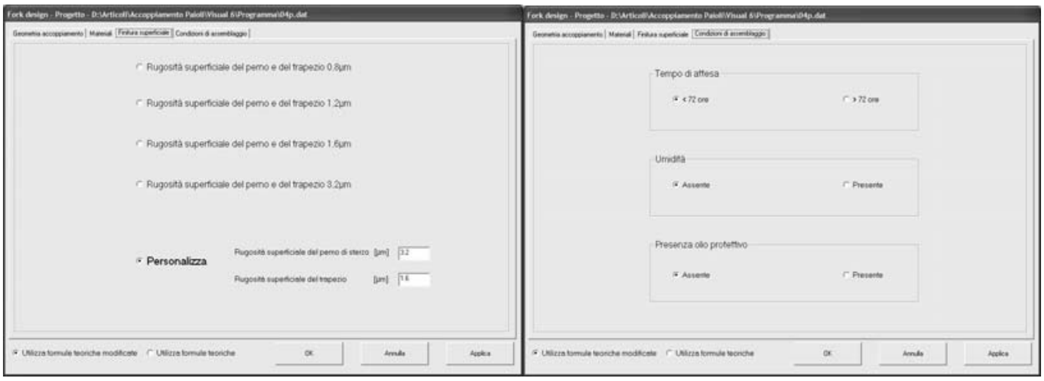

Figure 6. Input windows of the Fork Design program (design phase)

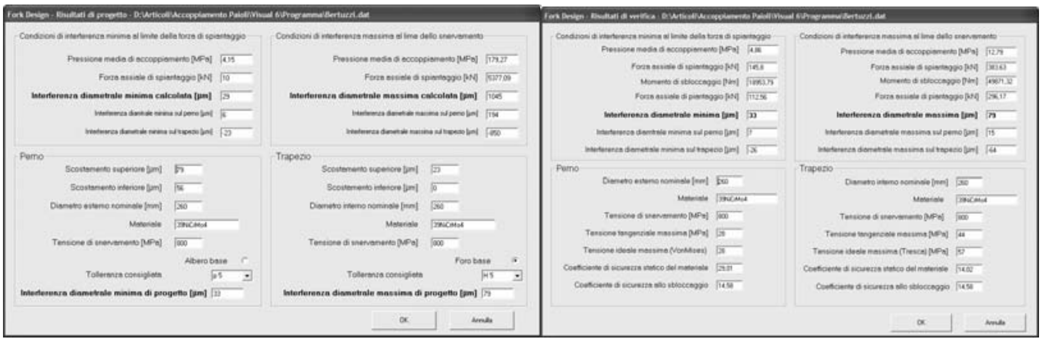

Figure 7. Output windows of the Fork Design program (design phase and verification phase)

5. Conclusions

The fundamental result of this work is the definition of a mathematical model which is useful for the design and the verification of joints with non axial-symmetric geometries. The discovered mathematical model, which corrects the theoretical formulas valid for the high thickness cylinders or axial symmetric geometries, has been applied in order to calculate the radial and the tangential mean tensions in fork-pin joints made both in steel and in aluminium by the Paioli Meccanica S.p.A.. Two corrective coefficients, β_r (2) and β_t (3), were calculated as ratios between the real tensions (radial and tangential in average) and the theoretical ones. The values of the real tensions have been calculated by performing some FEM analyses on different types of forks, while the theoretical ones applying the (8) and the (9) and considering the forks as cylinders with internal diameter Di_t and external diameter De_t=Di_t+2s (1). The final expressions have been defined taking into account all the data provided by the analyses and choosing the interpolation functions which minimize the errors. The errors of β_t are always lower than 5%, while those of β_r are always lower than 10%. The theoretical formulas (8) and (9) were then rewritten in (10) and (11) introducing the β_t and β_r coefficients. Finally, an original program was realised in order to perform the design and the verification of the fork-pin joints. This software may be successfully used in order to calculate the interferences of every fork-pin joint realized in steel or in aluminium, to define the critical value of the axial releasing force and to calculate the values of the interference on every coupled part. Furthermore, it is possible to find out, in a very short time, the minimum value necessary to overcome the releasing tests imposed by the law, and the range of the allowed releasing values in the assembly phase of components. Thanks to these results, it is possible to proceed completing the design of the joint also evaluating the local tensile state.

Possible future investigations concern the extension of these results to every kind of forks and, more in general, to other kind of shaft-hub joint which are not cylindrical or axial symmetric.

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Prof. Dario Croccolo PhD
DIEM – University of Bologna
Viale Risorgimento, 2 – 40136 BOLOGNA
Tel.: +390512093413
Fax.: +390512093413
Email dario.croccolo@unibo.it;
URL: http://www.diem.ing.unibo.it/personale/croccolo/index.htm