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CONSIDERATION OF SAFETY FACTORS FOR CYCLIC STRESSED MACHINE PARTS CALCULATED WITH FE METHOD - CASE STUDY

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1. Introduction

Reliable design of machine parts (power machines : turbines, generators, motors, fans, pumps etc.) is based of good knowledge of loads, material characteristics, environmental conditions and predicted useful life of machinery. To be competitive on world markets, finite-life (safe-life) design should not be applied only to airplanes but also to power machinery. Namely, machines based on infinite life design are more heavy, more costly and noncompetitive. Choise of adequate stress calculation methods, knowledge of material behaviour and adequate safety factors are essential steps in machine part design. Finite-life design is defined as life of machine part till crack initiation. High temperature machine components with creep behaviour will not be considered.

2. Stress and strain calculations

Power machine components (stationary and moving) are stressed with static loads (weight, assembly preloads); low cyclic loads (pressure, centrifugal forces, thermal stresses) and high cyclic loads (vibrations caused by rotation, media flow etc.). Loads can be nominal (operational - working pressure, nominal speed) or acceptional (over speed of rotors, test pressure, earth quakes etc.). In near past stresses and deformations were calculated with assumption of linear behaviour of materials and with simple analytical methods [VDI 2226, 1965], [DIN 18800, 1990], [Kostjuk 1982], [HTGD 41061, 1994] which gave as results “mean” or “avarage” stresses and deformations. There have been various improvements for calculaton of stresses : for stresses over yield point of material [VDI 2226, 1965] or for theoretical stress concentration factors K_t [VDI 2226, 1965], [DIN 18800, 1990], [Kostjuk 1982] witch were improved by introducing fatigue strength reduction factor K_f (called also fatigue notch factor) and Neuber rule for stresses and deformation in notches stressed over yield point [Tipton 1996].

Contemporary design of machines is based exclusively on CAD softwares (Autocad, Catia, Proengineer, etc.) and on Finite element methods (FEM)) of stress and strain calculation (Algor, Abacus, Nastran, Ansys, etc.). FEM calclulations are based on assumption of linear behaviour of material (FEM-L) or on non-linear behaviour of material (FEM-N). There are various methods and expressions for simulation of non-linear behaviour of materials. Most common are bi-linear [Tipton 1996], [Butković 2000] and Ramberg-Osgood [Tipton 1996], [Jelaska 2000]. Advantages of FEM-L calculations are : simple and fast calculations, exact values and distribution of stresses and deformation if stresses are within linear behaviour of material, possibility of calculations of thermal fields and thermal stresses. Disadvantages are: results are erroneous for stresses over yield strength of

material; values of stresses and strains contain theoretical stress concentration factor (K_t); there are possibility of errors if mesh is not adequate, and boundary conditions are not real.
To overcome problem of K_t stress can be corrected :

$$\sigma_k = \sigma_{FEM-L} \cdot \frac{K_{f,N}}{K_t} \tag{1}$$

where is : σ_k = corrected stress; σ_{FEM-L} = calculated stress; K_t = theoretical stress concentration factor; $K_{f,N}$ = fatique notch factor for N cycles.
If stresses are calculated with FEM-N, and are over yield point, instead of K_t, K_p has to be in (1).
Fatique notch factor $K_{f,N}$ is function of notch radius / grain size of material, stress level and number cycles to crack initiation [Collins 1993], [Tipton 1996]. For static load $K_{f,0.25}$ = 1 (for steels).
If calculated stress is over yield point, Neuber rule can be applied [Tipton 1996] (using expression for non-linear behaviour of material) :

$$\sigma_{FEM-L} \cdot \varepsilon_{FEM-L} = \sigma_k \cdot \varepsilon_k \tag{2}$$

where is : ε_{FEM-L} = calculated strain; ε_k = corrected strain.
Non-linear FEM calculations are entering technical praxis and advantages are : results of stress and strain calculations are exact for low and high stress levels (over yield point); thermal stresses can be calculated exactly, using nonlinear thermal and mechanical characteristics of material.
Disadvantages of FEM-N are : much more computer time and memory consuming; complicated preparations; possibly material data are not available; if stresses are lower than yielding point, results are the same as with FEM-L , but with much more computer time; stresses under yield point contain K_t (as in case of FEM-L) ; stresses over yield point contain plastic stress concentration factor (K_p) [Collins 1993].
To overcome problem of K_t, expression (1) should be used. Plastic stress concentration K_p is much closer to $K_{f,\infty}$ than K_t and can be assumed that $K_p = K_{f,\infty}$.
Bi-linear model is less accurate than other aproximation but for technical purposes bi-linear aproximations is satisfactorily and will be used in shown cases. Tenzors of stresses and strains can be presented by equivalent stress and strain. Von Mises eqivalency will be used [Collins 1993], [Tipton 1996], [Butković 2000]. Also static and cyclic part of stress have to be known (mean stress, stress amplitude, stress ratio) which is simply prepared from computed data.

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \tag{3}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{4}$$

$$r = \frac{\sigma_{min}}{\sigma_{max}} \tag{5}$$

where is : σ_m =mean stress; σ_a = amplitude of stress; r = stress ratio.

3. Properties of materials

Material data are needed for two purposes. First is to be able to calculate stresses and strains of loaded machine parts. For this purpose, if linear calculations are performed, only modulus of elasticity and Poisson-factor is needed for temperature range in which the machine parts operate. In the case of FEM-N (bi-linear) calculations more data are needed. For uniaxial relation of stress and strain valid is [Butković 2000] :

$$\begin{aligned} \varepsilon &= \frac{\sigma}{E} \quad (\sigma < R_{02}) \\ \varepsilon &= \frac{R_{02}}{E} + \frac{\sigma - R_{02}}{E_T} \quad (\sigma > R_{02}) \end{aligned} \tag{6}$$

where is : ε = strain; σ = stress; E = modulus of elasticity; E_T = strain hardening modulus of elasticity (tangential modulus); R_{02} = yield strength.

Second set of material data are strength oriented, as ultimate and yield strength, fatigue strength (endurance strength and low cyclic fatigue strength), for different number of cycles till crack initiation, for different stress ratios (static/cyclic) and for different temperatures, dimensions of parts; quality of surfaces, heat treatment of materials, corrosion influence of environment on material data etc. Most of data have to be achieved through testing on specimens, models or direct on machine parts, or can be derived from some of tested data. So, there is relation between endurance limit and ultimate strength [Butković 2000], and fatigue strength cyclic related and ultimate strength (see Figure 4). Some materials under cyclic straining change stress levels as cyclic softening and cyclic hardening or cyclic neutral material. In examples, only cyclic neutral material is considered, that means that monotonic and cyclic stress-strain curves are identical [Collins 1993], [Tipton 1996], [Butković 2000]. When stress are combination of static and cyclic, various theories for strength exist, as Sodeberg, Goodman, Gerber etc. [Collins 1993], [Tipton 1996], [Jelaska 2000], [Butković 2000]. It will be used Goodman theory, shown as Smith diagram, which is on safe side of calculation. Only stresses (not strains) will be used for assessment of safety.

4. Safety factors

Safety is not easy to define [Smith 1997], but can be described as relative protection from exposure to hazard, danger, failure, or crack initations (crack length 2 mm). The safety level depends on acceptable risk. Risk, on the other hand is not easy to measure (it always contain statistical aspects), because technical, economic, political, legal and moral issues are involved. Usually, safety is defined by state or international standards, codes or recommendations [VDI 2226, 1965], [DIN 18800, 1990]. In the case of large companies, with leading edge products there exist also internal safety codes [HTGD 41061, 1994], which comply with standards in the field of hazard.
Deterministic definition of safety factors is :

$$S = \frac{A}{B} \tag{7}$$

where is : S = safety factor for physical quantity and its state; A = ultimate value of physical quantity and its state; B = actual value of physical quantity and its state.

Physical quality can be stress, strain, deformation, force, number of cycles, time etc. State can be temperature, type of stress, stress ratio, time, etc. In technical praxis, a modified definiton of safety as safety margin is used :

$$M = \frac{S}{S_{all}} = \frac{A}{S_{all} \cdot B} = \frac{B_{all}}{B} \quad (8)$$

where is : M = safety margin; has to be ≥ 1 ; S_{all} = allowed safety factor; B_{all} = allowed actual value.

Statistical approach to safety [Rice 1997] is defined as allowed probability that machine will operate without failure in the service environment for the intended period of time. Further analysis of safety will be deterministic, but with values of material data with 0.99 probability of survival without failure (not 0.5). Also loads will be defined with 0.99 probability to be less than proposed.

Stress related safety in dimensioning in design is most common approach. Value of A is ultimate strength (R_m), yield strength (R_{02}), cyclic related strength (R_N), for various conditions of environment (temperatures), stress ratios etc. Value of B is Von Mises, maximal value of stress calculated with FEM-L or FEM-N method corrected with expression (1) or (2).

$$S_{s,y} = \frac{R_{02}}{\sigma_{max,ave}} \geq S_{s,y,all} \quad (9)$$

$$S_{s,N} = \frac{R_N}{\sigma_{max,k}} \geq S_{s,N,all} \quad (10)$$

where is : $S_{s,y}$ = stress-related safety factor for complete critical cross-section; $\sigma_{max,ave}$ = average (mean) stress values in cross-section for maximal load; $\sigma_{max,k}$ = corrected maximal stress; $S_{s,y,all}$ = allowed stress-related safety factor for average maximal stress; $S_{s,N}$ = stress-related safety factor for maximaly stressed material fabric; R_N = fatigue strength of material for N cycles and stress ratio; $S_{s,N,all}$ = allowed stress related safety factor for N cycles strength.

This means, that results of FEM calculations can not be directly applied, and recalculations are needed. Strain-related safety is similar to stress related safety, which has different values, but both values are interrelated with stress-strain curve. In the bi-linear model of material these relations are shown in equation (6). Ultimate total strain for steels is accepted as 4% (on safe side of calculation) because total fracture strain for design steels is 5% to 30%. If machine part is high prestressed and cycled influence of prestrain diminishes [Tipton 1996]. Deformation-related safety is mainly functionally related to machine parts and structures (rub of rotor in stator, leakege etc.). Load-related safety factor is defined in relation to loading capacity (force, moment or torque) as in elevators, bridges, chains, cars, lorries and aeroplanes and is not the same as stress-related safety. Only for bars evenly stressed both factors are equal. Safety factor cycle-related are defined:

$$S_c = \frac{N}{n} \geq S_{c,all} \quad (11)$$

where is : S_c = cycle related safety; N = number of cycles for physical quantity (stress, strain, force etc.) at conditions (temperature, frequency, stress ratio etc.); n = actual number of cycles on machine part; $S_{c,all}$ = allowed cycle-related safety factor.

At cyclic loading, it is often more convenient to take damage factor as:

$$D_c = \frac{n}{N} = \frac{1}{S_c} \tag{12}$$

In the case where the levels of cycle stresses are not the same during operating life, cumulative damage can be calculated on the basis of linear relations of damage (Miner rule [Collins 1993]) or same other relations [Collins 1993], [Tipton 1996]. Miner relation is:

$$D_c = \sum \frac{n_i}{N_i} = \sum \frac{1}{S_{c,i}} \leq \frac{1}{S_{c,c,all}} \tag{13}$$

Cumulative safety factor is reciprocite value of cumulative damage. There is possibility to define safety factors with other physical quantities as time, etc. Allowed values of safety factors are related to technical and scientific development of industry. First were used stress-related and load related safeties, based on linear simplified stress calculation and test loads.Vast numbers of standard and codes are issued [VDI 2226, 1965], [DIN 18800, 1990]. Generally, allowed values of safety depends on factors : data about material and loads, the application of the machine and how good the methods of calculation are in simulating physical phenomena, etc. For the case where the structure is stressed beyond the yield limit, standard and codes are not yet fully developed [Kostjuk 1982].
Factors of safety are different for personal elevators and for turbo-machinery. Shown are allowed safety factors for power machines, calculated with FEM-linear and non-linear method with correction mentioned in equations (1) and (2).
Stress related allowed safety factors are:

$S_{s,y,all} = 1.1 \div 1.7$ (equation (9) in relation to R_{02}),

$S_{s,N,all} = 1.1 \div 2.0$ (equation (9) in relation to R_N).

Cycle related allow safety factors are:

$S_{c,all} = 40 \div 200$ (equation (11) is related to N),

$S_{c,c,all} = 20 \div 100$ (equation (12) is related to Miner rule).

5. Examples

Two examples of calculation of machine parts design will be shown.

5.1 The last stage steam turbine rotor blades

Blades are free stading and are 800 mm long. The calculations of stresses and deformations (strains) were performed with program ALGOR, APAK 4, for rotor speeds from 0 to 6000 min^{-1}.
Basic material data for blade steel X20CrMoV12.1 at 50^0C are :
Yield point: R_{02}=580 MPa; Ultimate strength: R_L=750 MPa; Modulus of elasticity: E=0.22x10^6 MPa; Strain hardening modulus: E_T=4.4x10^3 MPa; Poisson ratio: μ=0.3; Specific weight: γ=7.7x10^6 N/mm^3.
A non-linear (bi-linear) FEM calculation was performed with force step increase method. Von Mises's stresses and strain levels, as well total blade deformations, were calculated. Figure 1. shows stresses in the blade air foil at 3000 min^{-1} , and in Figure 2. in the blade root at 3600 min^{-1} .
Figure 3. shows the results of stress calculations for two locations on the blade as a function of turbine speed.
Safety factors depend on the method of calculation, material data and load data. Material data (RL = 750 MPa) are minimum values with a probability higher than 0.99. Strain hardening modulus ET is taken greater than in reality, which gives results on the safe side. Fatigue strength values are derived from ultimate strength and are minimal values with a probability similar to ultimate strength (greater than 0.99). Only the centifugal force loading of the blade is taken into cosideration. Gas bending and high cyclic fatigue are neglected. The reason for this is low level of gas pressure stresses and the small probability that there would be resonance for first zi=1,…,8 harmonics of nominal

speed (fr = 50÷400 Hz). The turbine rotor life expectancy is calculated on 40 years (200000 operating hours). It is counted at 15 starts of the turbine in one year, which gives 40 x 15 = 600 low cycles for the blade at a nominal speed. It is counted at two overspeeds of turbine rotor a year, at 1.1 x nn = 3300 min-1 , which gives 40 x 2 = 80 low cycles from 0 to 1.1 x nn . It was calculated with two overspeeds to 1.2 x nn = 3600 min-1 in the lifetime of the blade.

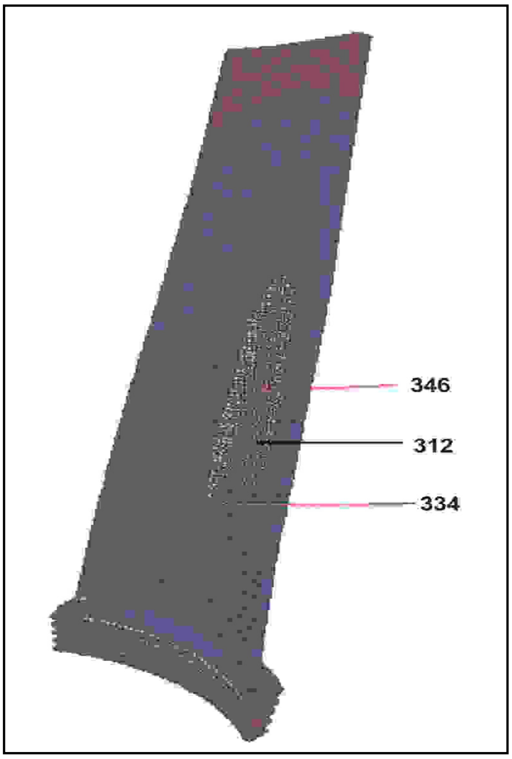

Figure 1.

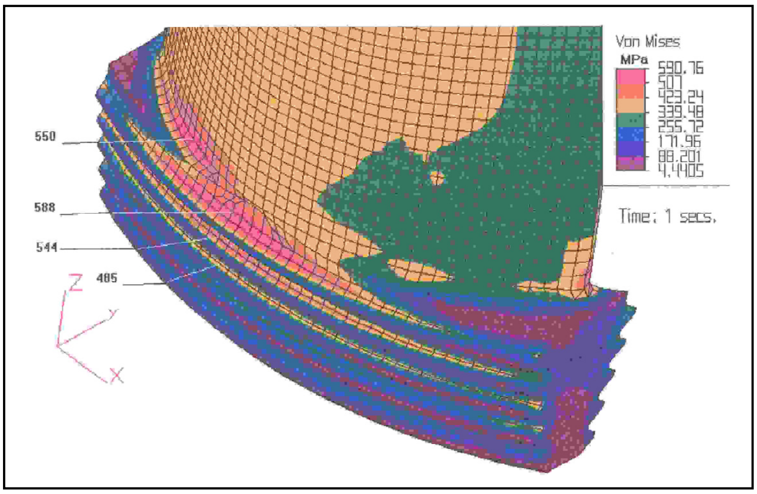

Figure 2.

During its lifetime, the blade is stressed with a pulsating type of low cyclic stresses, where mean and alternating stresses are equal, which values are half of the maximal stress.
Low cyclic fatigue data are shown in Figure 4., in a modified Goodman diagram (similar to the Smith diagram), where maximal total stresses are shown. Calculated blade stress points T_i are shown for two position on the blade and for nominal speed and for another two speeds (1.1 and 1.2 nominal speed).

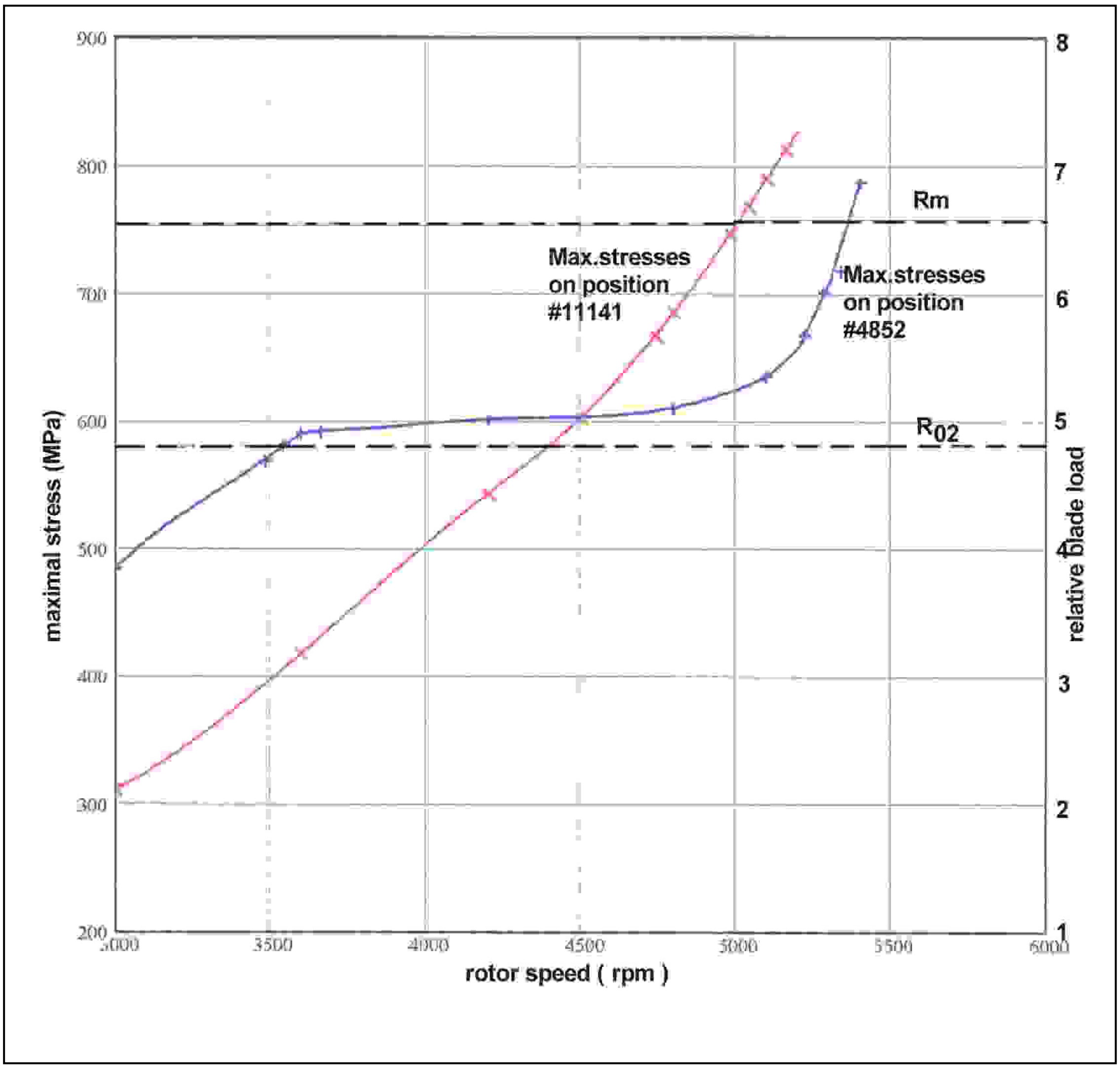

Figure 3.

Stresses are shown in Table 1. Points T_1 , T_2 , T_3 are on the root (not corrected) and $T_{1,k}$, $T_{2,k}$, $T_{3,k}$ are with corrected maximal stress. Points T_4 , T_5 and T_6 are stresses in blade air foil position 11141 , which are not the highest in air foil at a nominal speed, but will be highest when the stresses are higher than yield point which occurs at speeds higher than 4400 min^{-1} (see Figure 3.). Stress related safety factors are calculated for average stresses (expression (9)) and for maximal corrected and not corrected stresses (expression (10)), which are shown in Table 1. Correction of maximal stresses are performed with expression (1), where K_t for fir tree type blade root is 2.6 [Kostjuk 1982] and $K_{f,N}$ are taken from [Collins 1993]. All criteria for allowed safety factors are satisfied.

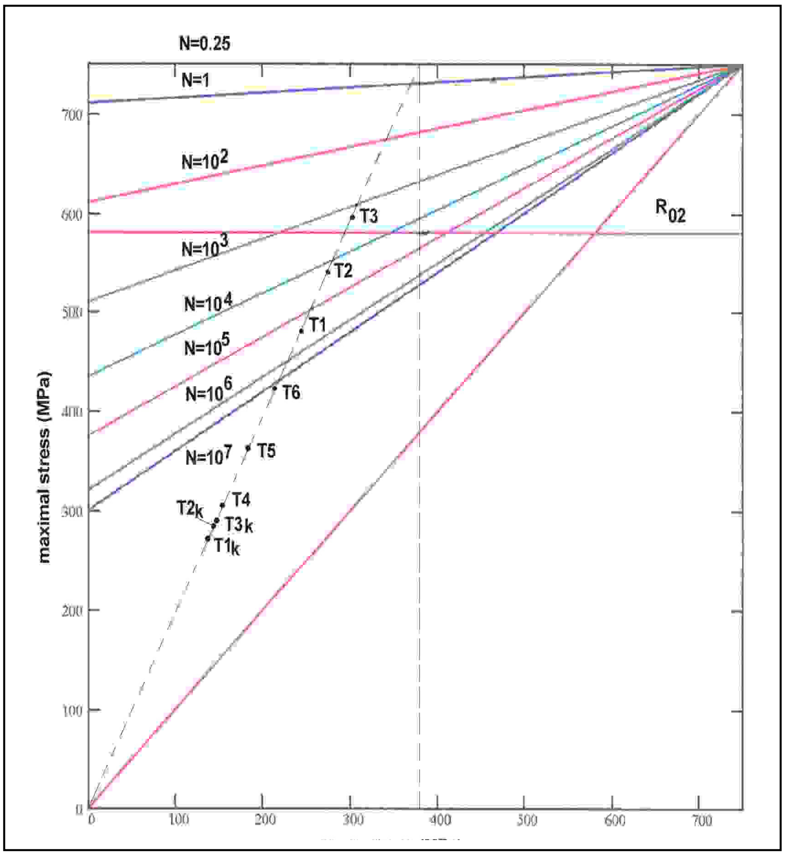

Figure 4.

Table 1. Safety factors and margins bor blade at position 4852 (root) and 11141 (air foil)

Position	Physical quantity	Rotor speed									Cumulative safety		
		nominal			1.1 x nominal			1.2 x nominal					
		σ_{ave}	R_{02}	$S_{s,y}$	σ_{ave}	R_{02}	$S_{s,y}$	σ_{ave}	R_{02}	$S_{s,y}$	S_c	D_c	M_c
		σ_{max}	R_N	$S_{s,600}$	σ_{max}	R_N	$S_{s,80}$	σ_{max}	R_N	$S_{s,2}$			
		$\sigma_{max,cor}$	$R_{N,c}$	$S_{s,600}$	$\sigma_{max,cor}$	$R_{N,c}$	$S_{s,80}$	$\sigma_{max,cor}$	$R_{N,c}$	$S_{s,2}$			
		n	N	$S_{s,600}$	n	N	$S_{s,80}$	n	N	$S_{s,2}$			
		n	N_{cor}	S_C	n	N_{cor}	S_C	n	N_{cor}	$S_{s,2}$			
1	2	3	4	5	6	7	8	9	10	11	12	13	14
4852	Average stress	91	580	6.37	110	580	5.27	131	580	4.42			
	Max. stress	484	601	1.24	546	665	1.22	588	745	1.27			
	Corrected max. stress	279	601	2.15	294	665	2.26	302	745	2.47			
	Cycles	600	300000	500	80	105000	131	2	2500	1250	96	0.01	4.8
	Cycles, correcterd	600	∞	∞	80	∞	∞	2	∞	∞	∞	0	∞
11141	Average stress	301	580	1.93	351	580	1.65	412	580	1.41			
	Max. stress	312	601	1.92	363	665	1.83	420	745	1.77			
	Corrected max. stress	312	601	1.92	363	665	1.83	420	745	1.77			
	Cycles	600	∞	∞	80	∞	∞	2	∞	∞	∞	0	∞
	Cycles, correcterd	600	∞	∞	80	∞	∞	2	∞	∞	∞	0	∞

5.2 Cooling fan hub coupling

Second example is calculation of safety for cooling fan hub coupling for combined cycle power station with finite life design of 7000 cycles (starts-stops) which material was proposed to be cast iron GGG 25-30. Material data are: $R_m = 300$ MPa, $R_{02} = 195$ MPa, E = 108 GPa, $E_T = 2749$ MPa.

Collar and shaft are fitted conicaly and keyed together. Fan is driven by electromotor, rated power 90 kW. Starting torque is $2.6M_r$. The fan operating speed is 73.38 RPM with nominal consumption of power of 0.6 rated ($M_n=0.6M_r$). Hub coupling is modelled of brick finite elements. Calculation is

performed with assumption that collar and shafts have middle interface fit (relatively light fit). For this case load is distributed half on the keyway and the half through contact friction forces. Key force is distributed on the key way surface in the trapeze form, with higher values on the shaft inlet side. For calculation ALGOR-FEM-non-linear (bi-linear) is used. Maximal stress is 312 MPa which is higher than R_m (it will be immediately cracked).
Recommended is new material for hub ST52-3N (DIN) or St-330S (W.Nr.1.11.33) (HN) with next characteristics : $R_m = 490$ MPa, $R_{02} = 275$ MPa, $E = 210000$ MPa , $E_T = 5556$ MPa .
Calculated stresses are :

Mean stress at maximal torque $\sigma_{max,ave} = 47$ MPa

Maximal stress with stress concentration $\sigma_{max} = 307$ MPa

Minimal stress $\sigma_m = 0$ MPa

Safety to mean (average) stress $S_{s,y} = \dfrac{275}{47} = 5.85$

Safety to fatigue related maximal stress (not corrected) $S_{s,7000} = \dfrac{R_{7000}}{\sigma_{max}} = \dfrac{380}{307} = 1.24$

and to corrected maximal stress $S_{s,7000,cor} = \dfrac{380}{201} = 1.89$

Safety cycles related (not corrected) $S_c = \dfrac{N}{n} = \dfrac{400000}{7000} = 57$

and for corrected stress $S_{c,k} = \dfrac{\infty}{7000} = \infty$

Material St52-3N satisfies all safety criteria and is recommended.

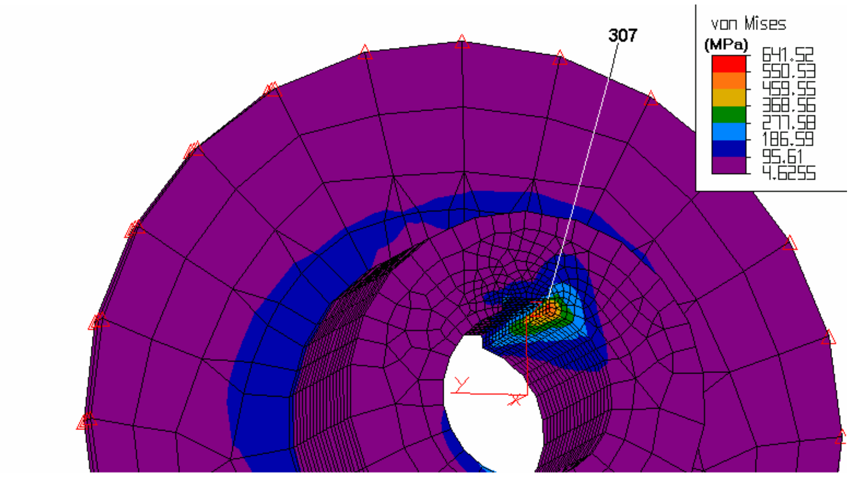

Figure 5.

6. Conclusion

The calculation of stresses using the non-linear (bi-linear) finite element is a big step forward in relation to linear methods of calculations for structures where stresses are higher than the yield point of the material. With the bi-linear FE method, more accurate values of stresses, strains and deformations are achieved than with any linear method.
Safety factors have to be related to average maximal stress and to maximal point stress with fatigue concentrations or to average maximal stress and to cycle-related critical point on the machine part, which demands additional recalculation for correction of stresses obtained with FEM calculations.

For large power machines design, recommended is finite-life concept of calculation with adequate safety factors.

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