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A FORMULATION OF COMPLEXITY-BASED RULES FOR THE PRELIMINARY DESIGN STAGE OF ROBOTIC ARCHITECTURES

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ABSTRACT
In this paper we propose a formulation capable of measuring the complexity of robotic architectures at the conceptual-design stage. The motivation lies in providing a tool to the robot designer when selecting the best alternative among various candidates generated at the early stages of the design process, when a parametric design is not yet available. While the performance evaluation of a robot includes many criteria, we focus on: the kinetostatic, the elastostatic and the elastodynamic performances; workspace volume; actuation complexity and the life-cycle cost. Within the realm of conceptual design, characterized by the absence of a mathematical model, it is not possible to optimize the performance at hand using classical mathematical programming methods. In this paper, a set of rules derived from robotics knowledge is outlined. These rules are then used to formulate a complexity measure used to filter-out less promising architectures at the conceptual stage. The complete formulation is applied to the development of a six-degree-of-freedom robot with low topological complexity, high performance and low actuation-system complexity. A complexity-comparison between the proposed architecture, the DIESTRO and the PUMA robots, is also provided.

Keywords: Conceptual design, robot design, complexity-based design

1 INTRODUCTION
Broadly speaking, the design process involves four main stages [1–4]: task definition; conceptual design; embodiment and detail design. At the conceptual design phase, concepts that satisfy the functional requirements of the desired product are identified and compared. It is said that approximately 75% of the total product life-cycle cost is committed in this phase [5]. The conceptual design phase has two essential sub-phases, namely, obtaining a rich solution set and short-listing the most promising solutions. The generation of the rich set pertains to the creative aspect of the design process. For this aspect, several techniques are available—brainstorming, synectics, TRIZ, and so on—but we will not dwell on these. Our work focuses on the selection sub-phase. The aim within this sub-phase is to minimize the number of concept variants and to reduce their chances of rejection at later stages. However, the solution to this problem is quite elusive, mostly because information about concept variants is scarce and rather qualitative at this stage. Notice that the main difference between a conceptual design and an embodiment is the absence of a *mathematical model* in the former.

Some tools have been developed over the years to help the engineer at the early design stages. These have been proposed in the form of principles that are applicable to all engineering design jobs, regardless of the discipline. Two main schools have contributed to the development of these tools: The German School and Axiomatic Design. The German School is highly developed, with proven guidelines approved and provided to the public by the VDI[1].

Axiomatic design [6] was proposed by MIT's Nam P. Suh (1990). Suh's paradigm is based on two main axioms, the independence and the information axiom. Several corollaries accompany these axioms. However, criticism on the pertinence of the Independence Axiom has appeared in the literature [7, 8].

[1]VDI is the acronym of *Verein Deutscher Ingenieure* (Association of German Engineers).

In this paper we try to improve the selection sub-phase of the conceptual design by improving the existing cost-benefit approach. In this vein, performance features against which concepts would be evaluated are established. We propose the use of complexity and entropy concepts to evaluate the complexity of various performance features. The design concepts are then improved based on the rule to improve performance. Weights are finally assigned to each performance feature and an overall complexity index is obtained which is suitable to compare designs.

2 COMPLEXITY OF KINEMATIC CHAINS

The kinematic chain being the skeleton of machines in mechanical engineering, and machines being a major source of design jobs, we dwell here on the complexity of these systems.

A kinematic chain is the result of the coupling of rigid bodies, called links, via *kinematic pairs*. When the coupling takes place in such a way that the two links share a common surface, a *lower kinematic pair* (LKP) results; when the coupling takes place along a common line or a common point, a *higher kinematic pair* (HKP) is obtained. Examples of higher kinematic pairs include gears and cams.

There are six LKPs, namely, revolute R, prismatic P, helical H, cylindrical C, planar F, and spherical S. The complexity of LKPs was evaluated in [9] by means of an index called the *loss-of-regularity* (LOR), inspired from Taguchi's *loss function* [10]. The LOR is a global index, as it measures how far a given surface lies from singularities. The LOR does so by measuring the spectral richness of the curvature changes of the surface under study.

The LORs of the six lower kinematic pairs, as reported in [9], are recorded in Table 1. The geometric complexity of these pairs is obtained by normalizing the mean LOR of each pair with respect to the maximum LOR, namely $LOR_{\mathrm{p}} = 19.6802$.

Table 1. Geometric complexity of the six lower kinematic pairs

Description	Loss of regularity			Geometric complexity
	male	female	mean	K_G
R	10.2999	10.2999	10.2999	0.5234
C	0	0	0	0
P	19.6802	19.6802	**19.6802**	**1**
H	15.8702	15.8702	15.8702	0.8064
F	7.6904	19.6802	13.6853	0.6954
S	0	0	0	0

Equipped with the LOR of the LKPs, the complexity of a kinematic chain can be evaluated in terms of the LKP used in the chain at hand and its corresponding LOR. One such formulation is suggested in this paper.

3 KINETOSTATIC, ELASTOSTATIC AND ELASTODYNAMIC PERFORMANCE

Kinetostatics is the study of the interplay between the feasible twists—point velocity and angular velocity—and the constraint wrenches—force and moment—in multi-body mechanical systems under static, conservative conditions. In robotic mechanical systems, a frequently used kinetostatic performance index is the condition number $\kappa \in [1, \infty)$ of the robot Jacobian $\mathbf{J}$ [11, 12], i.e.[2],

$$\kappa(\mathbf{J}) = \| \mathbf{J} \| \| \mathbf{J}^{-1} \| \qquad (1)$$

where $\| \cdot \|$ is a norm of $\mathbf{J}$. The Jacobian $\mathbf{J}$ of a robot is a matrix that maps the n-dimensional joint-rate vector $\dot{\boldsymbol{\theta}}$ into the six-dimensional twist $\mathbf{t}$ of the end-effector (EE). Additionally, $\mathbf{J}$ also relates the wrench $\mathbf{w}$ acting on the EE with the joint forces and torques $\boldsymbol{\tau}$ exerted by the actuators. The condition number of the Jacobian, representative of the distortion of these mappings, provides us with

[2]Robots for positioning and orienting tasks admit Jacobians with some entries with units of length and some that are dimensionless. Means to cope with this feature are available in the specialized literature [14], but are left aside here for the sake of conciseness.

a measure of how well the system behaves with regards to force and motion transmission. The Jacobian matrix is called *isotropic* when $\kappa(\mathbf{J})=1$ which represents the case when the mapping bears no distortion. A robot posture is called *isotropic* if it entails an isotropic robot Jacobian. A robot with at least one isotropic posture is called *isotropic* [12].

Elastostatic performance refers to the robotic-system response to the applied wrench under static equilibrium. This response may be measured in terms of the stiffness of the manipulator. The stiffness determines the translation and the angular deflection when the system is subjected to an applied wrench.

For serial robots, a simplified version to model robot deflection under static loading is commonly used. This model assumes that the links are rigid and that the joints are linearly-elastic torsional springs locked at a certain posture $\boldsymbol{\theta_0}$. The EE is subjected to a perturbation wrench $\Delta\mathbf{w}$ that is balanced by an elastic joint torque $\Delta\boldsymbol{\tau}$. Under these conditions, $\Delta\boldsymbol{\theta}$ and $\Delta\boldsymbol{\tau}$ obey

$$\mathbf{K}\Delta\boldsymbol{\theta}=\Delta\boldsymbol{\tau} \tag{2}$$

in which $\mathbf{K}$ is the *stiffness matrix* at the given posture.

For a constant magnitude of $\Delta\boldsymbol{\tau}$, the deflection attains its maximum value in the direction of the eigenvector associated with the minimum eigenvalue of $\mathbf{K}$, denoted by k_{min}. In terms of elastostatic performance, we aim at having (a) the maximum deflection a minimum, i.e., we want to maximize k_{min}, and (b) the magnitude of the deflection $\|\Delta\boldsymbol{\theta}\|$ as insensitive as possible to changes in the direction of the applied torque $\Delta\boldsymbol{\tau}$. This can be done by rendering k_{min} as close as possible to k_{max}. The first aim is associated with the stiffness constants, i.e., the higher the constants the lower the deflections. The latter, however, is associated with the concept of isotropy, the ideal case being when all the eigenvalues of $\mathbf{K}$ are identical, which means that $\kappa(\mathbf{K})=1$.

For a general design problem not only the kinetostatic and elastostatic performances have to be considered, but also the elastodynamic performance. In this regard, we introduce the foregoing assumptions, with the added condition that inertia forces due to the link masses and moments of inertia are now taken into consideration. The linearized model of a serial robot at the posture given by $\boldsymbol{\theta}_0$, if we neglect damping, is,

$$\mathbf{M}\Delta\ddot{\boldsymbol{\theta}}+\mathbf{K}\Delta\boldsymbol{\theta}=\Delta\boldsymbol{\tau} \tag{3}$$

in which $\mathbf{M}$ is the mass matrix of the robot expressed in the joint-space. Under "free vibration," i.e., under a motion of the system (3) caused by nonzero initial conditions and zero excitation, $\Delta\boldsymbol{\tau}=\mathbf{0}$, the foregoing equation can be solved for $\Delta\ddot{\boldsymbol{\theta}}$:

$$\Delta\ddot{\boldsymbol{\theta}}=-\mathbf{D}\Delta\boldsymbol{\theta},\quad \mathbf{D}\equiv\mathbf{M}^{-1}\mathbf{K} \tag{4}$$

with matrix $\mathbf{D}$ known as the dynamic matrix. This matrix determines the behavior of the system at hand, for its eigenvalues $\{\omega_i\}_1^n$ are the natural frequencies of the system and its eigenvectors $\{\mathbf{f}_i\}_i^n$ the modal vectors. The "harmonic response," of the system to an external excitation of frequency ω is known to have the frequency of the external excitation, i.e., ω, and a magnitude that depends on both ω and the frequency spectrum $\{\omega_i\}_1^n$ [13]. At resonance, i.e., when ω equals one of the natural frequencies of the system, the response magnitude grows unbounded. For this reason, when designing a robot, it is imperative that its frequency spectrum lie outside of the expected operation frequencies, which can be achieved by design.

4 THE FORMULATION OF COMPLEXITY-BASED RULES

At the conceptual stage, the designer has very limited information. The information typically includes the type, number and the relative arrangement of joints, along with the number of loops. Based on the functional requirements, the designer is usually able to decide on the type and the diversity of the actuators.

Three performance criteria were summarized in the last section, namely kinetostatic, elastostatic and elastodynamic. The designer would like to keep those concepts that are expected to perform well against the aforementioned criteria. Further, it is also desirable to keep the manufacturing, running and maintenance costs, or more generally, the *life-cycle* cost, at a minimum.

The kinetostatic performance depends on the robot Jacobian $\mathbf{J}$, which in turn depends on link dimensions and robot posture. Link dimensions are not available at the conceptual design stage, and hence, it is not apparent how a concept may be evaluated against the foregoing index at this stage. However, once a topology has been chosen, the kinetostatic performance can be optimized. The reader is referred to [14] for further details on this topic.

The elastostatic performance can be improved by increasing the stiffness of the robot structure. The elastodynamic performance, on the other hand, may be improved by increasing the stiffness, by decreasing the mass of the robot, or even by a combination of both. This increases the agility[3] of the robot. Hence, the best a designer can do to improve the elastodynamic and elastostatic performance at the conceptual design stage is to select topologies that have higher probability of being stiff and light in weight.

Most of the rules derived in the Subsection 4.1 are based on the foregoing discussion.

4.1 A Set of Design Rules

Table 2. The relation array between various performance criteria and the topology of a concept

	Number of joints	Number of loops	Type of joints	Joint configuration	Type of actuators	Diversity of actuators
Stiffness	R1.1	R1.2	-	-	R1.5	-
Life-Cycle Cost	R2.1	R2.2	R2.3.1 R2.3.2	R2.4	R2.5	R2.6
Workspace Volume	-	R3.2	R3.3.1 R3.3.2	-	-	-
Agility	R4.1	R4.2	-	-	R4.5	

R1.1 The number of joints in a robot should be minimized to increase stiffness.

LKPs, i.e., revolute, prismatic, etc., are known to introduce compliance in the robot structure, in the same way that they do in machine tools [15]. Constraint forces within a joint assembly are typically supported by a reduced area that is under high stress, which results in high strains. Increasing the contact area is one way to obtain stiffer joints. However, this approach would result in an increase in the mass of the joint and hence, in that of the robot. However, from the agility point-of-view, increasing the mass of the robot is not desirable.

Hence, using the correct type and size of joints is imperative, which calls for a trade-off between the stiffness and the mass of the joint[4].

Of course, this conflict can be avoided if the joint is removed altogether. Based on the above discussion, the probability of a kinematic chain to be stiff is higher if less joints are used. In fact, if the base frame is connected to the EE frame 'directly,' i.e., without joints in-between, the 'chain' is the stiffest possible, but it would loose its functionality.

R1.2 Increasing the number of loops has a minor impact on the stiffness of the robot

Although increasing the number of loops generally increases the stiffness of a robot, while doing so, extra degrees of freedom must be introduced to assemble the robot, which significantly reduces the gain in the stiffness.

R1.5 Electromagnetic actuators are more compliant than hydraulic actuators

The torque applied by an electromagnetic actuator is proportional to the current passing through it. Hence, to make the motor stiffer, more energy must be dissipated. Notice that this energy is dissipated

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[3]By agility we refer to the property of a robot to achieve high and accurate operational speeds; speeds are usually measured in terms of cycle times, for industry-adopted standard cycles.

[4]In the authors' opinion, it is common practice in research circles to spend the lion's share of the budget on high-quality motors and control system, while leaving little for high-quality joints

in the form of a RI^2 loss in the motor that could overheat the armature. Therefore, a compromise on the stiffness of an electromagnetic system is unavoidable.
On the other hand, the stiffness of hydraulic actuators depends on the compressibility of the fluid. Since hydraulic fluids are virtually incompressible, hydraulic actuators are known to exhibit high stiffness.

R2.1 Increasing the number of joints increases the manufacturing cost
The manufacturing and the maintenance cost of any product is directly proportional to the number of parts used in it, other factors assumed equal.

R2.2 Increasing the number of loops increases the manufacturing cost
Increasing the number of loops typically requires additional degrees of freedom to allow for assembly, and hence, additional joints.

R2.3.1 The six lower kinematic pairs are, in order of decreasing preference: cylindrical, spherical, revolute, screw, planar, prismatic
This rule stems directly from Section 2.

R2.3.2 Revolute joints are easiest to maintain
As revolute joints are compact and much easier to seal, they demand less maintenance.

R2.4 Increasing the diversity in geometric constraints between joints increases the manufacturing cost
This rule is true from the machining as well as from the inspection point-of-view. For example, machining only parallel or only perpendicular bores is generally more cost-effective than machining a combination of the two. By the same token, the inspection equipment employed to verify these constraints decreases with a decrease in the foregoing diversity.

R2.5 Electromagnetic actuators have a lower life-cycle cost than their hydraulic counterparts
Hydraulic actuators need additional equipment, such as a hydraulic pump, a reservoir, etc. This brings additional initial cost into the system. These systems also have higher maintenance costs.

R2.6 Increasing the actuator diversity increases the cost of the robot
Both the manufacturing and the maintenance costs increase with an increase in actuator diversity. Here, diversity refers to both type and size. Notice that due to the *pyramidal effect* of serial robots, in which downstream motors carry their upstream counterparts, the diversity of serial robots is higher than their hybrid or parallel counterparts.

R3.2 Increasing the number of loops can only decrease the workspace volume
At the conceptual stage, the workspace volume we refer to is *dimensionless*. Of course the workspace volume may be increased or decreased by appropriately scaling the link lengths of the robot. However, these lengths are not available at the conceptual stage, for which reason this rule warrants further explanation, which we give by means of an example: consider two RRR serial chains. If the revolutes are arranged appropriately, each manipulator is known to have a positioning workspace of the shape of a sphere [12]. Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be the workspaces of the two RRR manipulators; then, if the EEs of the two serial chains are welded, a 2RRR parallel manipulator with a workspace $\mathcal{S}_1 \cap \mathcal{S}_2$ is obtained. Apparently, $\mathcal{S}_1 \cap \mathcal{S}_2$ cannot have a greater volume than that of any of $\mathcal{S}_1$ or $\mathcal{S}_2$.

R3.3.1 A Revolute joint at the base of a serial robot is desirable for an axially symmetric workspace
This rule is based on the simple way of generating axially symmetric surfaces in geometric modeling, i.e., by a simple revolution operation.

R3.3.2 A Prismatic joint at the base of a serial robot is desirable for workspaces with extruded symmetry
Ditto for the extrusion operation.

R4.1 Increasing the number of joints decreases agility
Increasing the number of joints increases the mass. The reason behind is that good quality, stiff joints are usually heavy. Hence, the agility of the robot is directly affected by increasing the number of joints.

R4.2 Addition of loops to allow actuator(s) placed closer to the base increases agility
The mass of a robot can be significantly reduced by moving the actuators closer to the base, thereby increasing the robot agility. One way to do this is by the use of concentric tubes and bevel gears, as in the TELBOT System [16]. Besides having no dead load on its link by virtue of the motors, TELBOT has unlimited angular displacement of its joints and no cables traveling through its structure. However, this construction introduces inaccuracies from two sources, backlash between gears and high compliance due to long concentric tubes that behave as compliant torsional springs.
Another way to place the actuators closer to the base is by introducing additional loops, the architecture thus changing from serial to parallel. Notice, however, that R4.2 reaches its threshold when all the actuators have been placed on the base. Adding more loops thereafter would not increase the robot agility.

R4.5 In robotics, the use of hydraulic actuators increases agility
Hydraulic actuators have higher power-to-size ratio as compared to their electromagnetic counterparts. Hence, their use would reduce the mass of the robot, thus increasing its agility. Notice, however, that R2.5 conflicts with R4.5 and hence, a trade-off is unavoidable.

4.2 Complexity-Based Rules
We define the complexity of a robot based on the rules outlined in the previous section: a robot architecture should be minimally complex if it abides by the above rules. Below we define six aspects of robot complexity:

4.2.1 Joint-Number Complexity K_N
The joint-number complexity K_N is defined as:

$$K_N = 1 - \exp(-q_N N) \tag{5}$$

where N is the number of joints used in the topology at hand and q_N is the *resolution parameter*, to be adjusted according to the resolution required. Note that $K_N \in [0,1]$.

4.2.2 Loop Complexity K_L
Notice that R1.2 and R4.2 conflict with R2.2 and R3.2. In this vein, the designer must provide the minimum number of loops l_m ; l_m could be the minimum number of loops required to produce a special displacement group or subgroup [17]. The loop complexity K_L of a robot is defined as:

$$K_L = 1 - \exp(-q_L L); \quad L = l - l_m \tag{6}$$

where l is the number of kinematic loops in the topology of the robot.

4.2.3 Joint-Type Complexity K_J
Joint-type complexity K_J is that associated with the type of LKPs used in a kinematic chain. We define this complexity as

$$K_J = \frac{1}{n}(n_R K_{G|R} + n_P K_{G|P} + n_C K_{G|C} + n_F K_{G|F} + n_S K_{G|S} + n_H K_{G|H}) \tag{7}$$

where n_R, n_P, n_C, n_F, n_S and n_H are the numbers of revolute, prismatic, cylindrical, planar, spherical and helical joints, respectively, while n is the total number of pairs and $K_{G|x}$ is the geometric complexity of the pair x as recorded in Table 1.

4.2.4 Link Diversity K_B

At the conceptual design stage, partial information about the geometric relations between neighboring joints is available. However, this partial information suffices to allow us to distinguish five possible link *topologies* (Figure 1), as the relative layout between its two associated joint axes defines a binary link[5] .

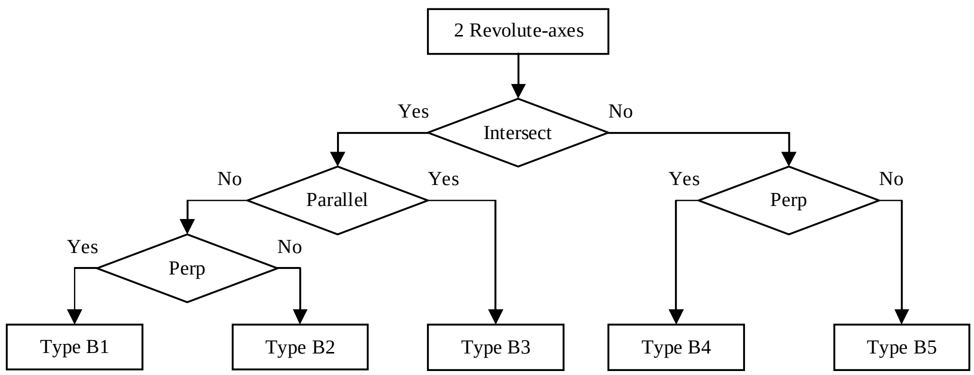

Figure 1. Binary tree displaying possible link topologies

We borrow the concepts of entropy from molecular thermodynamics and from information theory [18] to help us evaluate the effect of geometric-constraint diversity, at the conceptual stage. In this vein we define the geometric-constraint diversity as:

$$K_B = \frac{B}{B_{max}} \tag{8}$$

where B is the entropy of the link topologies and B_{max} is the maximum possible value of B. We thus have

$$B = -\sum_{i=1}^{c} b_i \log_2(b_i); \quad b_i = \frac{M_i}{\sum_{i=1}^{c} M_i} \tag{9}$$

in which c is the number of distinct joint-constraint types used in a concept and M_i is the number of instances of each type of joint-constraints. Moreover, $B = B_{max}$ when all the above five constraint types are used with equal frequency, i.e., $B_{max} = \log_2(5) = 2.32$ bits.

4.2.5 Actuator-Type Complexity K_A

R2.5 is in conflict with R1.5 and R2.5; hence, a provision to resolve this conflict must be provided in the formulation. The actuator-type complexity is defined as:

$$K_A = 1 - \exp(-q_A A); \quad A = a - a_m \tag{10}$$

where a is the number of electromagnetic actuators in the robot topology at hand, while a_m is the minimum number of electromagnetic actuators allowed.

[5]Ternary and higher-order links can be accommodated, but we will leave the discussion of these aside in the interest of brevity. As well, we assume only revolute joints in this brief discussion.

4.2.6 Actuator-Diversity K_H

The concept of entropy can be used again to evaluate the effect of actuator diversity. We define the actuator-diversity, termed here actuator-complexity, as:

$$K_H = \frac{H}{H_{max}} \qquad (11)$$

where H is the entropy of the set of actuators and H_{max} is the maximum possible value of H, attained when no two actuators are identical.
We thus have

$$H = -\sum_{i=1}^{d} p_i \log_2(p_i); \quad p_i = \frac{N_i}{\sum_{i=1}^{d} N_i} \qquad (12)$$

in which d is the number of distinct actuator types or sizes and N_i is the number of instances of each type or specification. Moreover, if $N \equiv \sum_1^d N_i$, then $H_{max} = \log_2(N)$ [19].

4.2.7 Definition of the resolution parameters

Three resolution parameters, namely, q_N, q_L, and q_A were introduced above. These parameters provide an appropriate resolution for the complexity at hand. Since the foregoing formulation is intended to compare the complexities of two or more kinematic chains, it is reasonable to assign a complexity of 0.9 to the chain with maximum complexity, and hence, evaluate the normalizing constant, i.e., for $J = N, L, A$,

$$q_J = \begin{cases} -\ln(0.1)/J_{max}, & \text{for} \quad J_{max} > 0; \\ 0, & \text{for} \quad J_{max} = 0. \end{cases}$$

4.3 The Total Complexity of a Robot Kinematic Chain

Finally, we define the complexity $K \in [0,1]$ of a kinematic chain as a *convex combination* [20] of its various complexities:

$$K = w_N K_N + w_L K_L + w_J K_J + w_B K_B + w_A K_A + w_H K_H \qquad (13)$$

where w_N, w_L, w_J, w_B, w_A and w_H denote their corresponding weights, such that

$$w_J + w_N + w_L + w_B + w_A + w_H = 1$$

These weights must be assigned by the designer based on the type of functions for which the robot is designed.

5 EXAMPLE: A SIX-DOF HYBRID ROBOT

In this section we propose a hybrid six-dof robot (C1) and compare it the PUMA (C2) and the DIESTRO (C3). DIESTRO [12] is a six-axis isotropic manipulator.

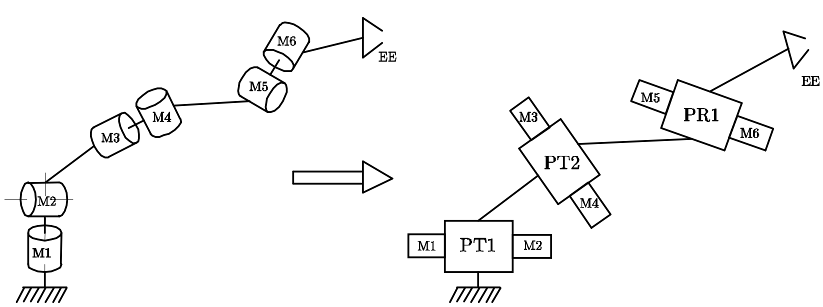

Figure 2. Proposed 6-dof hybrid manipulator

Figure 2 shows the skeleton of the proposed six-dof hybrid robot. The robot is ‘hybrid’ because it is a concatenation of three parallel subsystems, namely two pan-tilt (PT) and one pan-roll (PR) mechanism. Both, the pan-tilt and the pan-roll mechanisms substitute a set of two mutually perpendicular revolutes in series, as shown in the figure. Two *identical* motors drive each pan-tilt (pan-roll) mechanism. Linear combinations (difference and mean value) of the angular velocities of the two motors provide the pan and the tilt (roll).

Table 3 displays, the DH-parameters of the three manipulator concepts at hand. Notice that in Table 3 we have recorded only the information that is available at the conceptual stage, and left out the joint variables, as these are irrelevant to our discussion.

Table 3. The DH-parameters of the three concepts at hand

	C1			C2			C3		
Joint	a_i	b_i	α_i	a_i	b_i	α_i	a_i	b_i	α_i
	m	m	deg	m	m	deg	m	m	deg
1	0	0	90	0	b_1	90	a	a	90
2	0	0	90	a_2	b_2	0	a	a	-90
3	0	b_3	90	a_3	0	-90	a	a	90
4	a_4	0	0	0	b_4	-90	a	a	-90
5	0	0	90	0	0	90	a	a	90
6	a_6	b_6	α_6	a_6	b_6	α_6	a	a	α_6

Tables 4 and 5 display all information required to calculate the various complexities discussed in Section 4.

Table 4. Information available at the conceptual design stage

	l	l_m	n_R	n_P	n_C	n_F	n_S	n_G	a	a_m			
C1	0	0	6	0	0	0	0	0	6	6			
C2	0	0	6	0	0	0	0	0	6	6			
C3	0	0	6	0	0	0	0	0	6	6			

	d	N_1	N_2	N_3	N_4	N_5	N_6	c	B_1	B_2	B_3	B_4	B_5
C1	3	2	2	2	-	-	-	2	4	0	1	0	0
C2	6	1	1	1	1	1	1	3	3	0	1	1	0
C3	6	1	1	1	1	1	1	1	0	0	0	5	0

Table 5. Parameters required to compute the complexities

	N	L	B	A	H
		$l-l_m$	$-\sum_{i=1}^{c} b_i \log_2(b_i)$	$a-a_m$	$-\sum_{i=1}^{d} p_i \log_2(p_i)$
C1	6	0	0.722	0	1.585
C2	6	0	1.371	0	2.585
C3	6	0	0	0	2.585
q_J	0.383	0	-	0	-
Max. entropy	-	-	2.322	-	2.585

All types of the associated complexities of C1, C2 and C3 are recorded in Table 6, from which apparently, K_B and K_H are different for the three concepts. Hence, we use only these two complexity types to compare the concepts under study. Assuming an equal weight of $w_B = w_H = 0.5$, we obtain $K_{C1} = 0.462$, $K_{C2} = 0.795$ and $K_{C3} = 0.5$.

Notice that the complexity of C2 is always greater than that of C1 and C3 regardless of the distribution of weights. However, neither C1 nor C3 is 'globally' superior from the other. For our case, we would like to give more weight to the actuator diversity in the hope of improved agility. We thus select C1 for detail design.

Table 6. Complexities of the two concepts under study

	K_N	K_L	K_J	K_B	K_A	K_H
C1	0.9	0	0.523	0.311	0	0.613
C2	0.9	0	0.523	0.590	0	1.0
C3	0.9	0	0.523	0	0	1.0

6 CONCLUSIONS

This paper proposes a formulation to evaluate the complexity of various robots at the conceptual stage. In this vein several rules were outlined and six complexity indices were proposed. The total complexity was found by assigning weights to each type of complexity. A comparison between three concepts, namely, a six-dof hybrid manipulator, the PUMA 560 robot and the DIESTRO robot, was also provided.

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