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SMOOTH CURVES AND MOTIONS DEFINED AND OPTIMISED USING POINT-BASED TECHNIQUES

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1. Introduction

Smooth motions are important in many areas of engineering, including robotics and machine design [Röschel 1998]. Typically there are conflicting requirements and the design task can be formulated as a constrained optimisation problem. Examples of requirements include: smooth motion (good curvature profile), minimal time of travel, peak acceleration and jerk (third derivative) below prescribed limits (to reduce inertial forces and damage to product), and obstacle avoidance.
A common way to specify and improve motions is in terms of Bézier and B-spline techniques which are based upon parametric representations. Optimisation schemes are applied to their control points in order meet performance goals. However, each control point affects a significant portion of a motion and may not give the fine-tuning needed for some applications. Additionally, the parametric definition can lead to problems as the parameter is only a means for defining a motion and does not itself determine the geometry. For these reasons it can be easier to deal with actual points along the motion and use variation of these to achieve performance requirements and functional constraints.
In general, a large number of points is required in order to provide an adequate representation and this increases the numerical complexity considerably. However, it is felt that computing power (and the available algorithms) currently available mean that this is no longer the problem it once was, and so it is worth investigating the applicability of point-based techniques for design optimisation. The next section reviews parametric descriptions and then section 3 introduces the point-based approach. It emerges that there is a requirement to preserve, through any design optimisation process, the regularity of the points, and the continuity and smoothness of the curve they "define". Three strategies for doing this are proposed in section 4 and evaluated by means of case study examples.

2. Smooth curves and motions

There are a number of ways for describing smooth curves and motions. For example, Bézier and B-spline techniques are well known means for defining and manipulating free-form curves. Each curve is defined as a parametric function $\mathbf{r}(t)$ in which the parameter varies over a range of values. The function itself is a linear combination of pre-specified control points which are combined with "weights" which are functions of the parameter t. This allows some meaning to be assigned to the control points in terms of the geometry of the curve itself. For example, the following is the form for the Bézier cubic segment

$$\mathbf{r}(t) = \mathbf{a}_0(1-t)^3 + 3\mathbf{a}_1 t(1-t)^2 + 3\mathbf{a}_2 t(1-t)^2 + \mathbf{a}_3 t^3 \quad (1)$$

in which the parameter t runs between zero and unity. Then the curve segment starts at the first control point $\mathbf{a}_0$ and ends at the last $\mathbf{a}_3$. The initial tangent is in the direction of the vector ($\mathbf{a}_1$-$\mathbf{a}_0$), and the end tangent in the direction of ($\mathbf{a}_3$-$\mathbf{a}_2$).

By varying the control points, the form of the curve can be modified. Bézier segments can be created of any degree. However high degrees can lead to unwanted oscillations. Instead it is preferable to join together individual segments of low degree, and this is effectively what the B-spline formulation achieves.

Whilst smooth curve segments can be used to define motion paths [Röschel 1998], it may additionally be required that the motion defines the orientation of a body as it moves. This requires the introduction of rigid body transforms, that is transforms which are a combination of a rotation and a translation. Means for dealing with such transforms include the use of quaternions and geometric (Clifford) algebra [Fang et al. 1998, Etzel & McCarthy 1999]. It is then possible to extend Bézier and B-spline techniques to allow combinations of transforms. Here prescribed control transforms are combined instead of control points [Mullineux, 2004]. Figure 1 shows an example of a cubic motion.

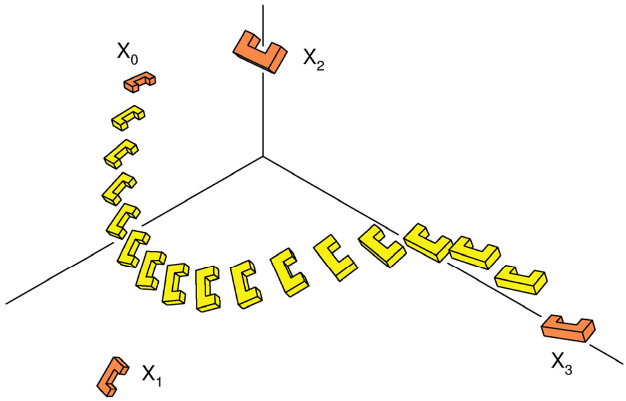

Figure 1. Bézier cubic motion

The use of such techniques provides control points which govern the shape of the curve or motion. Given a suitable measure of performance for the motion, it can be straightforward to apply a suitable optimisation process to find a "best" version. For example, figure 2 shows an example of a pick-and-place robot used to transfer packs from a conveyor into a packing case for shipment. The end-effector is carried at the end of a moving beam whose motion is controlled by driving the other end along a vertical track and a point at its centre along a horizontal track. A combination of the motions of these two points allows the end-effector to reach any position within a region of space. In designing the motion for a particular task, there is a requirement to keep the motion smooth, minimise the peak acceleration (to avoid damaging or dropping the product), and to avoid obstacles (such as the side of the case or other packs already in it). For any path, the maximum acceleration along it is assessed and a penalty introduced if it passes through the obstacle. An optimisation strategy is then used to adjust the control points to minimise this performance value. If a fifth degree Bézier curve is used, with fixed end points, then there are four intermediate controls points (eight degrees of freedom) which can be adjusted. Shown on the right of figure 2 are the resulting motions for varying sizes of an obstacle.

3. Point-based approach

The advantage of dealing with the parametric form is that the number of control points is reasonably small and so there is little opportunity for numerical search procedures to become unstable. However, there are some drawbacks. Firstly, the adjustment of a single control point can affect a large part of the curve shape; in fact with a Bézier segment, moving any control point affects the entire segment to some extent. This means that is may not be straightforward to fine-tune a profile. An example of this can be seen in the third case on the right of figure 2 where the curve turns in at its lowest point rather than the shortest route towards this end. A second drawback is that, as entities are defined parametrically, unwanted effects, such as oscillations, can occur if too high a degree is used. Thirdly, the parameter is not a true geometric property: it merely represents the speed at which the curve segment is traced out. As such, there is the difficulty of trying to optimise the true geometry via an intermediary. This has also been observed in the use of these techniques within geometric modelling systems [Cripps 2003].

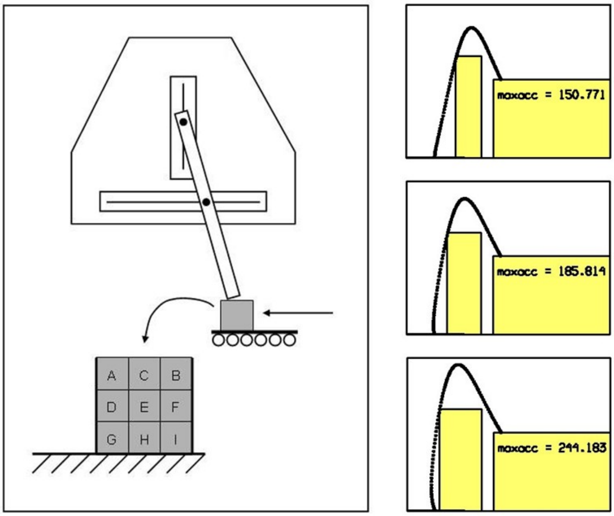

Figure 2. Pick-and-place operation and path optimisation

An additional drawback arises when it is necessary to apply constraints to particular parts of a motion curve. For example, figure 3 shows the motion created by a four bar transfer mechanism producing a roughly triangular path. The application here is to transfer goods from one conveyor to another moving at right angles. It is necessary to apply constraints to achieve particular velocities at two specified points in the cycle corresponding to where the goods are picked up and set down. The figure shows passage from an initial configuration, through variation of the geometry to a final acceptable motion.

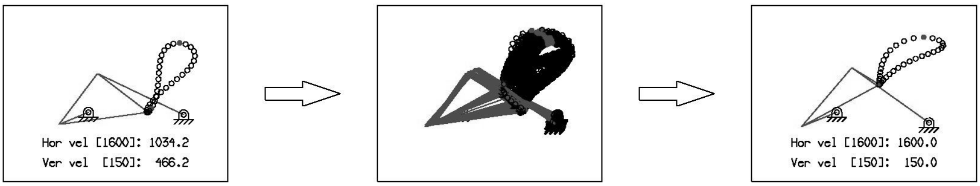

Figure 3. Improvement of motion of a transfer mechanism

In order to address these limitations, an alternative approach discussed here that deals with the representation of the curve and motion as sequences of points. This allows constraints and performance measures to be applied to regions and to individual points. This also has the advantage that it is the actual geometry that is being considered.

There is of course a need to deal with a much larger number of degrees of freedom and hence with a more complex system. However, it is felt that (with care) standard optimisation procedures are able to cope [Feldman et al. 2008]. An area in which this approach has been applied successfully is that of improving the aesthetic design of a surface [Mullineux 2002]. For example, the left part of figure 4 shows a surface patch (which is roughly an octant of a sphere) represented as a collection of individual points. These are associated in the sense that each has a prescribed set of neighbours. The quality of the surface is assessed by evaluating the surface (Gaussian) curvature at each point and applying constraints to ensure that this varies smoothly. When this is done and the internal points are allowed to vary, the improved patch shown on the right of the figure is achieved.

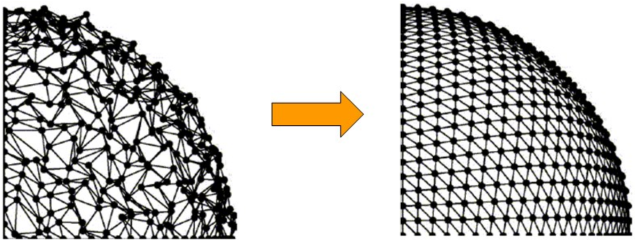

Figure 4. Optimisation of surface quality

However, the approach is limited. Consider the example shown in figure 5. Here it is desired to design a path that avoids two obstacles shown as the two shaded rectangles. A similar point-based strategy is applied in which a performance function is used based on the total length of the path defined as a sequence of points. Penalties are added into the function if points lie within the prohibited regions. The figure shows the result of one attempt to vary the sequence of points so as to minimise the performance function. What has happened is that the implied relationship between adjacent points has not been preserved. This happens naturally when a parametric curve definition is present, but needs to be maintained explicitly if a point sequence is used.

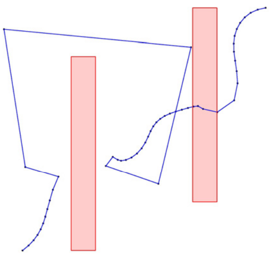

Figure 5. Improper path variation

4. Three strategies

In conventional design optimisation [Papalambros & Wilde 2000], a number of design parameters are identified. These are then allowed to vary in a search for an optimal design. Effectively as each is varied, the corresponding partial derivative of the performance measure is considered; the larger this value, the greater the sensitivity.

In designing a motion path, it is effectively necessary to vary a whole function (rather than individual values) which is more akin to the aims of the calculus of variations. By representing the path as a sequence of points, the approach considers only individual values, but there is a need to preserve the fact that they are related as a sequence derived from a function. In computational terms, it is highly desirable to be able to deal with an "object" (which could be a sequence of points) which has an attribute of being changed in certain acceptable ways which preserve the sequential properties. Additionally, the ability to increase or decrease the number of points representing the path is desirable. In searching for a good design for a motion path, one approach is initially to use a fairly coarse representation with few points, and then to increase the number as the search moves closer to an optimal design configuration. In order to preserve regularity, continuity and smoothness of a motion path, three strategies are proposed and examples of each are given in the following subsections. The first is to allow only design searches which move points (approximately) normal to the current motion path. The second is to allow general movement of points but then to apply techniques to improve regularity and smoothness. The third is to introduce (into the performance evaluation) constraints which ensure that the points generate a smooth path.

4.1 Normal moves

The first strategy is only to employ search steps in which an individual point moves normal to the current motion path. However moving just a single point naturally compromises the smoothness of the new trial path. So instead the entire point set is adjusted. To provide a framework to do this, points are moved (normal to the current curve) through (small) distances which vary sinusoidally along the length. This way the end points can be kept fixed. The search movements can be regarded as Fourier harmonic components of more general displacements.

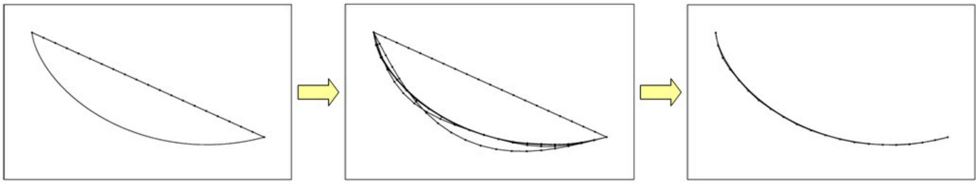

Figure 6. Brachistochrone problem

Figure 6 shows this approach used to solve the classical brachistochrone problem which is to find the shape of wire to connect two points in space such that a bead sliding under gravity along it takes the minimum time to drop from the higher to the lower. The exact solution is an arc of a cycloid. The left part of the figure shows a straight line used as the initial path; the curve is the required cycloid. The second part shows the use of Fourier moves to minimise the time of travel. Just applying the first harmonic gives a good approximation to the exact solution and this is then refined. The last part of the figure shows the final path obtained which matches the cycloid almost perfectly.

However the approach has drawbacks. Figure 7 shows the same approach used to try to design a smooth path between two points which avoids a given obstacle. Again the initial path is a straight line, and variation using the first harmonic gives the curved path shown. This succeeds in avoiding the obstacle, but clearly its length could be reduced. However, further attempts to try harmonic moves fail. This is because they either enter the obstacle (thus incurring a penalty) or move away from the top of the obstacle (thus increasing the path length). This drawback could be overcome by allowing more general search steps, however by themselves these are unlikely to preserve the smoothness of the path.

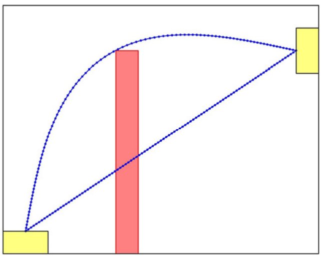

Figure 7. Fourier-based based for obstacle avoidance

4.2 General moves and smoothing

More general search moves are possible provided that the resultant path can be smoothed. This subsection discusses an approach to provide such smoothing [Mullineux & Robinson 2007]. Given a smooth curve "defined" by a sequence of points, the curvature at each can be approximated using finite difference techniques. This allows a graph of curvature against arc length, the curvature profile, to be obtained. This is normally "spiky" because of the numerical approximations involved. It is straightforward to smooth this profile, for example by replacing each value by a weighed average of its neighbours.

The "original" curve can now be recreated by working from one end of the curvature profile and integrating twice. Such integration also improves smoothness. However, in general the point so derived for the other end of the curve does not meet its required target. Although it is possible to adjust the points (angular position) and the arc length progressively along the curve so that the derived end point matches the original. This entire process is carried out again, starting from the other end of the point sequence. This gives two new curves and a weighted average (dependent upon distance from the starting end) of the two is then taken as the curve with improved smoothness. Since the integrations are performed numerically, the resultant points are uniformly spread along the arc length, and their number can be increased or decreased as required.

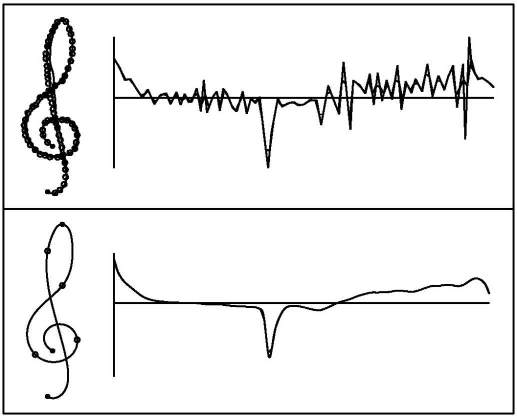

Figure 8. Improving curve smoothness using curvature plot

An example of this approach is given in figure 8 which uses the shape of a musical G-clef. The upper part of the figure shows the initial shape obtained by digitising points. The corresponding curvature profile is far from smooth. When smoothing is applied (over four iterations), the result is that in the lower part. Here the number of points required to define the shape has been much reduced and the curvature profile is smoother. It is necessary to ensure that the topmost point is regarded as fixed to ensure that the whole shape does not collapse to a single point. To allow search trials to be undertaken it is necessary to increase the number of points along the curve to more than shown here.

4.3 Smoothness constraints

The third strategy is to introduce constraints to ensure that smoothness is preserved. This can be done, for example, by modifying the performance measure for a motion path so as to penalise it when curvatures become large. This approach is illustrated by reference to an example arising from the design analysis of packaging equipment [Hicks, et al. 2003]. Carton board is commonly used to form secondary packaging for foodstuffs and other products. The board is a laminate material and is supplied to industrial packers a flat, pre-cut and pre-creased, blanks. To "erect" a carton, machine operations are designed to fold the board along the creases. The creases have weakened the material and during folding the material delaminates resulting in a fold that has a reduced tendency to spring back. The design process for the machine requires understanding of the material properties and this can be obtained by modelling the delamination process.

Figure 9 show the three plies of a board represented as rows of points. During the analysis, one end of these rows is held fixed and the other is translated and rotated. It is desired to find what delamination occurs. The performance function used is created by assessing three form of “energy” within the system. The first is that due to bending; this is assessed by the equivalent of normal beam theory and as such includes a representation of the curvature and hence the smoothness. The second is the energy due to longitudinal extension or compression of each layer. The third is transverse compressive energy (across the three layers); extension across the layers is not counted as this represents delamination.

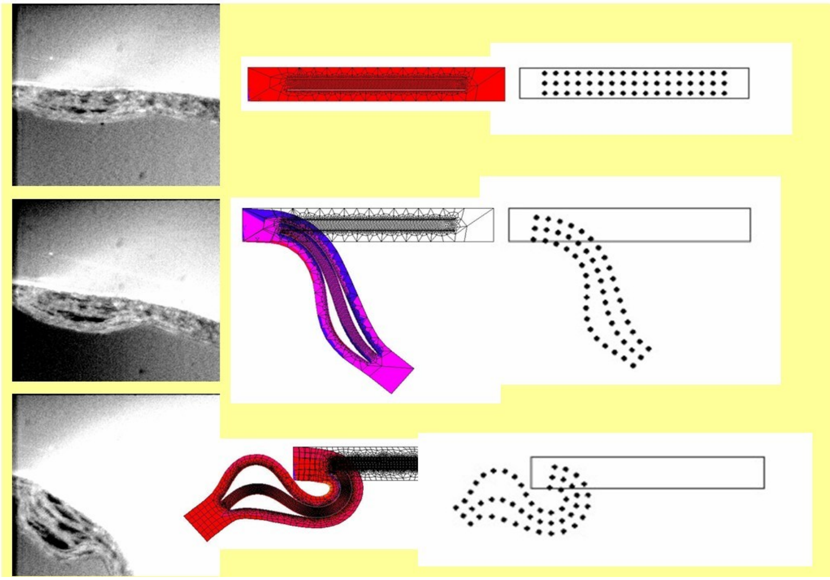

Figure 9. Point-based model of delamination

The free end of the layers is then moved over a number of steps and for each one, the configuration of minimal energy is found by allowing all the points (except at the ends) to move generally. The results at two steps are shown in figure 9. Also shown are the results of a finite element (FE) analysis (together with photos of corresponding delamination of actual board). The two modelling results are in good agreement which shows that the introduction of the smoothness constraints is effective. Although not of concern here, it was also found that the point-based approach produced results more quickly than the use of the general purpose FE package.

5. Conclusions

There is need in several aspects of engineering design to be able to deal with smooth curve and motions. One way, perhaps the conventional way in CAD systems, is to impose some form of definition via a parametric function. This allows the entire shape to be dealt with as a single entity and for it to be adjusted be means of a small number of control values or points. This allows design optimisation to be achieved based on a suitable measure of performance.

However this may restrict the amount of searching that can be undertaken and additionally introduces parameters which may not be directly related to the geometry of the situation. This paper has looked at the alternative which is to treat the curve or motion as being described by a sequences of points which can then be varied individually. This certainly adds to the complexity of the design optimisation but has the advantage that it is the actual geometry that is being considered and a wider variety of possible solution forms is available.

It is important to ensure that continuity, smoothness and regularity of the points are preserved while the design search progresses. It is suggested that this can be achieved in three ways. The first is by only allowing searches which move points normal to the current curve. The second is by allowing more general moves but then adjusting the point set to a more continuous, smoother form. The final way is to introduce additional constraints to the design optimisation which ensure that the required conditions continue to be met. All three have been found to work well for particular applications. However, the simplicity of the first approach means that searching can be too limited and a fully optimal configuration may not always be achieved. This is overcome with the other two approaches which, although they are computationally more expensive, do enable a more complete searching of possible paths and motions to be undertaken.

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