



## OPERATING BOLT LOAD CALCULATION IN JOINTS OF COMPLEX ELASTIC STRUCTURES

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### 1. Introduction

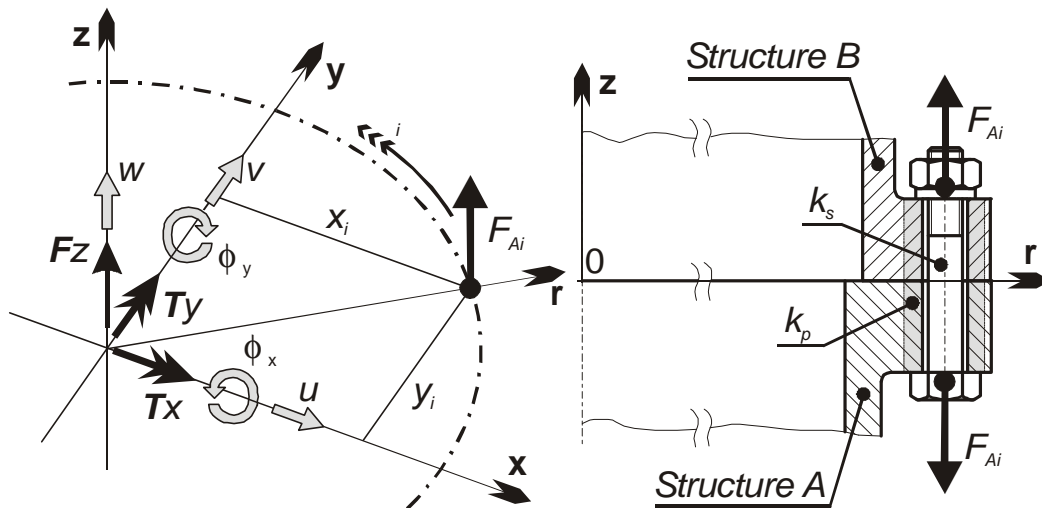
Procedures for the calculation of pre-stressed and dynamically loaded bolts are nowadays theoretically well known, practically tested and proved and also prescribed with different standards, regulations and recommendations [*Richtlinie VDI 2230 2001*]. Unfortunately, this is not the case with the calculation of true bolt operating loads. The issue is especially important at certain critical joints with evident uneven stiffness of supporting structures in the bolt vicinity and/or with uneven bolt distribution. In rotational connections, for example, bolted joints are used to fix rings of large-sized rolling bearings onto supporting structures. The effect of uneven stiffness of supporting structures on the external load distribution on the rolling elements of a bearing is an important issue also in bearings. Our group has researched [*Prebil 1994, Prebil 1995*] these effects. The findings have been included into an improved model to determine the maximum operating force  $F_A$  in individual bolts in a joint [*Prebil 1998*]. The generalised model is based on the use of so-called elasticity and stiffness matrices of both structures joined by a bolted joint. In the continuation, the paper briefly presents the model for operating bolt load calculation and the method for the calculation of structure elasticity and stiffness matrices and presents a comparative analysis of the results obtained through different calculations and measurements.

### 2. Definition of the algorithm

The model for operating bolt load calculation (Fig. 1) is based on the following definitions, presumptions and generalisations:

- A bolted joint joins two structures, A (supporting) and B (upper), which are made of Hook's ideal elastic material. All the deformations and displacements are small and remain in the elastic domain.
- The entire bolted joint is located on one plane (the joining plane of two flanges). On this plane lies the x-y plane of a right-handed Cartesian co-ordinate system so that the positive +z axis points towards structure B (upper). The position of the co-ordinate system origin is not prescribed but it is only logical for it to be located in the center of gravity of the bolt distribution pattern.
- The external forces and moments which affect the bolted joint act upon structure B and travel through a bolted joint onto structure A which is fixed (supported) in space. External loads are notated with the resulting force acting in the direction of the described co-ordinate system (Fz) and with the resulting moment acting around x (Tx) and y (Ty) axes of the co-ordinate system. Bolts in a joint are only loaded in the axial (z) direction.

- In calculation of the bolt operating force we take into account only elasticity deformations and those displacements in bolts, flanges and structures which run in the z direction of the co-ordinate system, that is in the axial direction of the bolts.
- The number of bolts in a joint is n and their distribution pattern is not prescribed. Bolt position is generally determined with the co-ordinates  $x_i$  and  $y_i$ , which are stated numerically as input data. Bolt position, i.e. the co-ordinates record, is usually presented in mathematical form with vector notation.
- It is necessary to know or at least roughly estimate the dimensions of a bolted joint which are required in calculation of elasticity ? or stiffness k of joint bolts and flanges (1).



**Figure 1. Bolted joint model (bolts are distributed along circular path in this case)**

The elasticity of bolts and flanges (1) along with the corresponding forces, as defined in VDI 2230 and presented in Fig. 1, is the only elasticity taken into the account at classical methods of computation. In the expression,  $l$  is the length (of a bolt or flange),  $A$  is the surface of cross-section, and  $E$  the elastic modulus of material.

$$d = \frac{f}{F} = \frac{l}{AE} \rightarrow F = \frac{1}{d} f = k \cdot f \quad (1)$$

The load and reaction forces on Fig. 1 are used to set up a system of three equations representing the static equilibrium of structure B:

$$F_z - \sum_{i=1}^n F_{Ai} = 0, \quad T_x - \sum_{i=1}^n F_{Ai} y_i = 0, \quad T_y + \sum_{i=1}^n F_{Ai} x_i = 0 \quad \text{and} \quad F_{Ai} = F_{si} - F_{pi}. \quad (2)$$

Index  $s$  is used to represent the bolt and index  $p$  is used to represent the flange (Fig. 2). The bolt operating force is positive if it applies additional tension to the bolt, and negative if it causes additional compression of the flange (reduces bolt load). The force operates at positions where the elasticity of a bolt and flange and the performance of external loads cause a relative displacement (as of a stiff body) of structure B towards structure A.

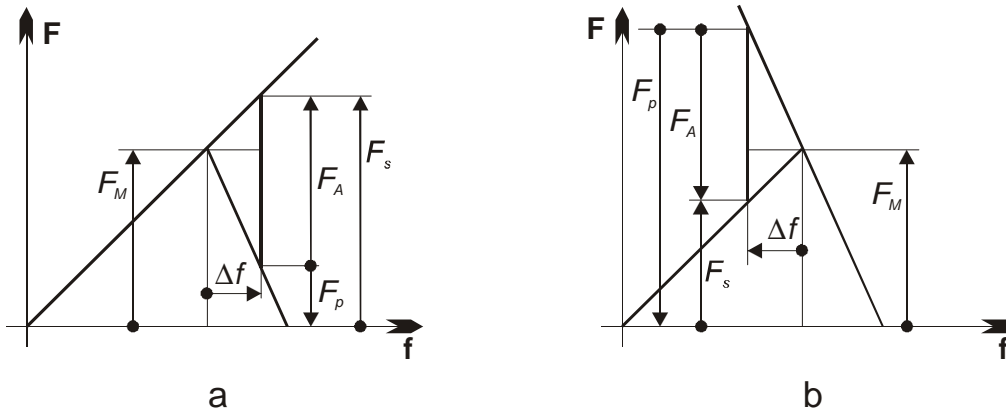
The shift of structure B is mathematically described with a geometric transformation of the co-ordinate system, which consists of its rotation about the y and x-axes ( $f_y, f_x$ ) and the translation of  $u, v$  and  $w$  along x, y and z-axes (Fig. 1). Since the rotation angles are very small, equation can be simplified as follows ( $\sin f \rightarrow f$  [rad],  $\cos f \rightarrow 1$ ):

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{f}_x \mathbf{f}_y & \mathbf{f}_y & u \\ 0 & 1 & -\mathbf{f}_x & v \\ -\mathbf{f}_y & \mathbf{f}_x & 1 & w \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}. \quad (3)$$

According to the mentioned presumptions, an elasticity deformation of a bolt and flange is only caused by rotations around the x and y-axes and a shift towards the z-axis. In this type of connections, position vectors running from the co-ordinate system origin to the bolts are relatively long, especially if compared with the clamping length of bolts or flange thickness. In such cases the elastic extension of a bolt or flange can be written in a simplified form as

$$\Delta f_i = z_{1i} - z_i = -\mathbf{f}_y x_i + \mathbf{f}_x y_i + z_i + w - z_i = -\mathbf{f}_y x_i + \mathbf{f}_x y_i + w. \quad (4)$$

If the elastic extension is positive, it results in an increased bolt load and a decreased flange load, and vice versa if the elastic extension is negative (Fig. 2).



**Figure 2. Additional bolt (a) and flange (b) load**

On the basis of expressions (1) and (4) and the bolted joint loads diagram (Fig. 2), the expression to determine the operating force on individual bolts can be written as

$$F_{Ai} = (k_s + k_p) \Delta f_i = (k_s + k_p) \cdot (-\mathbf{f}_y x_i + \mathbf{f}_x y_i + w). \quad (5)$$

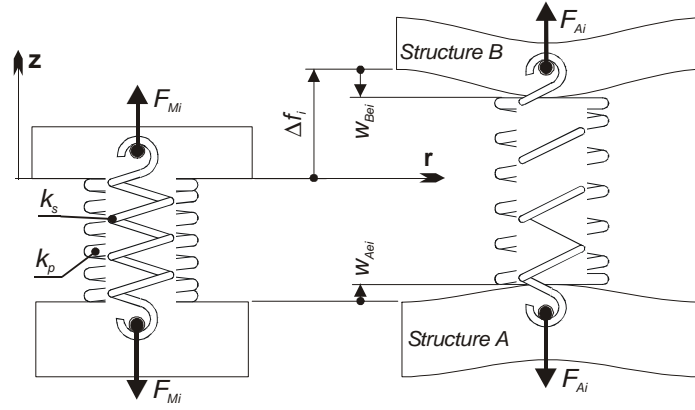
Symbols  $k_s$  and  $k_p$  represent the stiffness [N/mm, N/m] of a bolt and flange respectively as defined in equation (1).

### 3. Role of the elasticity of joined structures

External load operation on structure B causes the operating force  $F_{Ai}$  to be applied to the bolts of elastic structures A and B joints. Besides causing extensions and contractions of bolts and flanges, these forces also result in elastic displacements of both structures at bolt positions, which in turn affect the sizes of operating forces  $F_{Ai}$  because of uneven elasticity (stiffness) of structures.

In accordance with the definitions stated in Chapter 2 and on the basis of Fig. 3 we can deduce that a positive operating force  $F_{Ai}$  causes in structure A a positive elastic displacement  $w_{Aei}$  in the z-axis direction, and in structure B a negative elastic displacement  $w_{Bei}$  in the z-axis direction. The vectors of elastic reaction forces of structures A and B upon the vector of operating force  $F_A$  can be presented as:

$$\{Q_{Ae}\} = \{F_A\}, \quad \{Q_{Be}\} = -\{F_A\}. \quad (6)$$



**Figure 3. Elastic displacements of structures A and B resulting from the operating force  $F_{Ai}$**

Since the structures A and B are continuous, the operating force  $F_{Ai}$  affects elastic displacements of structures at position  $i$  and at all other bolt positions, and vice versa. The relation between elastic forces of structures and elastic displacements at bolt positions can be summarised in the following matrix equations:

$$\{Q_{Ae}\} = [K_A] \cdot \{w_{Ae}\}, \quad \{Q_{Be}\} = [K_B] \cdot \{w_{Be}\}, \quad \{Q_{Ae}\} = -\{Q_{Be}\} \quad (7)$$

Matrices  $K_A$  and  $K_B$  represent the stiffness [N/mm, N/m] of structures A and B at bolt positions. The matrices are square matrices  $n \times n$  in size, where  $n$  is the number of bolts in a joint. Later in the paper more shall be said about how to calculate these matrices for given structures.

Elastic structure displacements ( $w_{Aei}$ ,  $w_{Bei}$ ) affect the extensions and contractions of bolts and flanges at their positions and, consequently, the actual operating force size. A bolt operating force at position  $i$  can in this case be presented as:

$$F_{Ai} = (k_s + k_p) \cdot (\Delta f_i - \Delta f_{ei}) = (k_s + k_p) \cdot (-\mathbf{f}_y x_i + \mathbf{f}_x y_i + w - w_{Aei} + w_{Bei}) \quad (8)$$

From the equations in (7) a relation between elastic displacements of structures A and B can be derived and written as:

$$\{w_{Ae}\} = -[K_A]^{-1} \cdot [K_B] \cdot \{w_{Be}\} = -[P_A] \cdot [K_B] \cdot \{w_{Be}\} = -[P_A K_B] \cdot \{w_{Be}\} \quad (9)$$

Expressions (6) and (7) can be used as the basis for the equilibrium system of  $n$  equations:

$$[K_B] \cdot \{w_{Be}\} + \{F_A\} = 0 \quad (10)$$

Expressions (8) and (9) help us obtain the final matrix notation of the equilibrium system (10):

$$\left( \frac{1}{k_s + k_p} [K_B] + [P_A K_B] + [I] \right) \cdot \{w_{Be}\} - \mathbf{f}_y \{x\} + \mathbf{f}_x \{y\} + w \{1\} = 0 \quad (11)$$

System (11) has  $n$  equations and  $n+3$  unknowns (elastic displacements  $\{w_{Be}\}$  and body displacements  $\mathbf{f}_y$ ,  $\mathbf{f}_x$ ,  $w$ ). The missing three equations are obtained from the equilibrium with external loads (2), which can be presented in matrix form as follows:

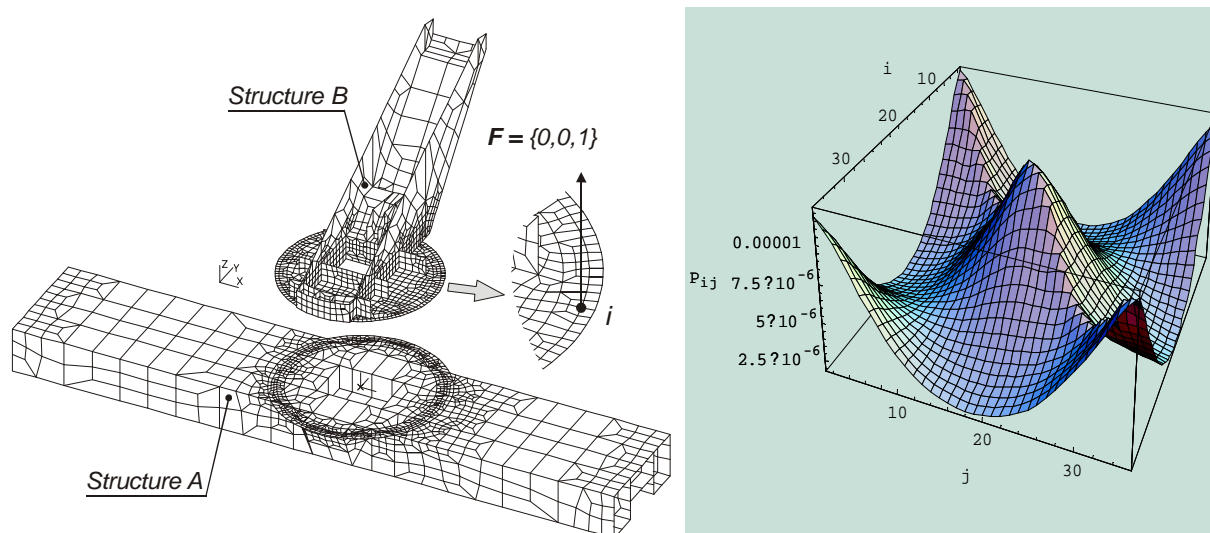
$$\begin{aligned} (k_s + k_p) \left( \left( [P_A K_B] + [I] \right) \cdot \{w_{Be}\} \right)^T \cdot \{y\} - \mathbf{f}_y \{x\}^T \cdot \{y\} + \mathbf{f}_x \{y\}^T \cdot \{y\} + w \{1\}^T \cdot \{y\} &= T_x, \\ (k_s + k_p) \left( \left( [P_A K_B] + [I] \right) \cdot \{w_{Be}\} \right)^T \cdot \{x\} - \mathbf{f}_y \{x\}^T \cdot \{x\} + \mathbf{f}_x \{y\}^T \cdot \{x\} + w \{1\}^T \cdot \{x\} &= -T_y, \\ (k_s + k_p) \left( \left( [P_A K_B] + [I] \right) \cdot \{w_{Be}\} \right)^T \cdot \{1\} - \mathbf{f}_y \{x\}^T \cdot \{1\} + \mathbf{f}_x \{y\}^T \cdot \{1\} + w \{1\}^T \cdot \{1\} &= F_z. \end{aligned} \quad (12)$$

The size of all matrices is  $n \times n$  and they are written in square brackets. Column vectors of  $n$  lengths are written in wrapped brackets, while superscript  $T$  represents the mathematical operation of transposing a matrix. The expression  $\{a\}^T \{b\}$  is therefore a scalar product of vectors  $a$  and  $b$ .  $[I]$  is the unit matrix,  $\{1\}$  is the unit vector. The linear system of  $n+3$  equations (11 and 12) can be solved with a suitable numeric algorithm (e.g. LU decomposition) or with a suitable software tool for symbolic and numeric calculation (e.g. Mathematica). The next step is to use equation (8) to calculate the actual size of operating forces at all bolts.

### 3.1 Elasticity and stiffness matrices of joined structures

The presented model is relatively simple for it enables a more accurate calculation of bolt operating loads with a reasonably easy method for solving the linear system of equations. Nevertheless, it requires calculation of the elasticity and stiffness matrices of structures A and B for bolt positions in structures (defined in the 3D Cartesian co-ordinate system). The task can be solved using any suitable method for linear elastic structure analysis. Modern engineering practice recommends the use of a commercial CAE package for FEM (Finite Element Method) analyses [SAS 1998].

Structures (A and B separately) need to be modelled accurately by using appropriate structural linear elastic elements (beam, shell, solid – Fig. 4). It is necessary that the mesh in the joint vicinity is sufficiently dense and that there are nodes at bolt positions. For practical reasons it is also advisable to logically enumerate the nodes at bolt positions (ascending sequence reflecting bolt distribution – e.g. along a circular line). When conducting an elasto-static analysis, structure A (supporting) is supported at positions and in the way that imitates the actual supports. Structure B (upper) is supported at positions where the structure is affected by external loads, and in the way it is affected.



**Figure 4. FEM model of mobile crane (shell FEM elements) and elasticity matrix caculated for 36 bolt positions on structure A**

For each structure the  $n$  load cases ( $n$  is the number of bolts) are prepared. The  $i$ -th load case is the unit force  $\{0,0,1\}$  in the  $+z$  direction, which is applied at the node in the mesh where the  $i$ -th bolt in the structure is positioned. We are only able to use the results of the analysis to create an elasticity matrix of the structure ( $[P_A], [P_B]$ ) if we calculate for each load case the displacements of the structure  $\{w_{Xe}\}$  in the  $z$ -axis direction for all nodes where bolts are located. Every element  $P_{ij}$  [mm/N, m/N] of an elasticity matrix of structure represents a displacement in the  $z$  direction at position  $j$ , which is caused by the application of unit force in the  $z$  direction at position  $i$ . The stiffness matrix of the studied structure is calculated by inverting ( $K=P^{-1}$ ) [Wolfram 1993, Press 1990] elasticity matrix of this structure. Every element  $K_{ij}$  [N/mm, N/m] of the stiffness matrix of structure represents the reaction force in the  $z$  direction at position  $j$  caused by unit displacement in the  $z$  direction at position  $i$ .

Provided the model is carefully designed and the results obtained with an FEM analysis are logical, elasticity matrix inversion should not be a difficult task. If the nodes at which displacements are calculated are logically enumerated, every row in an elasticity matrix represents discrete values of the displacement curve of the structure in the z direction along a connected path on which bolts are distributed (often a circle, a rectangle, ...). A 3D representation of such a matrix is a continuous surface with dominant diagonal elements (Fig. 5). These matrices are well conditioned and invert easily.

#### 4. Advantages of the method

A calculation of the elasticity and stiffness matrix is quite time consuming. We need to build an FEM model which is sufficiently accurate, carry out an analysis for a considerable number of load cases and perform a relatively extensive research into the analysis results by using suitable macro programs of the CAE package or specially designed applications in the suitable programming language. Although most of these tasks can be automated to a high extent, the question of whether and when the use of the described method is sensible in comparison with the classic FEM analysis of the entire joined structure is well justified. Our experience shows that the presented method has the following main advantages:

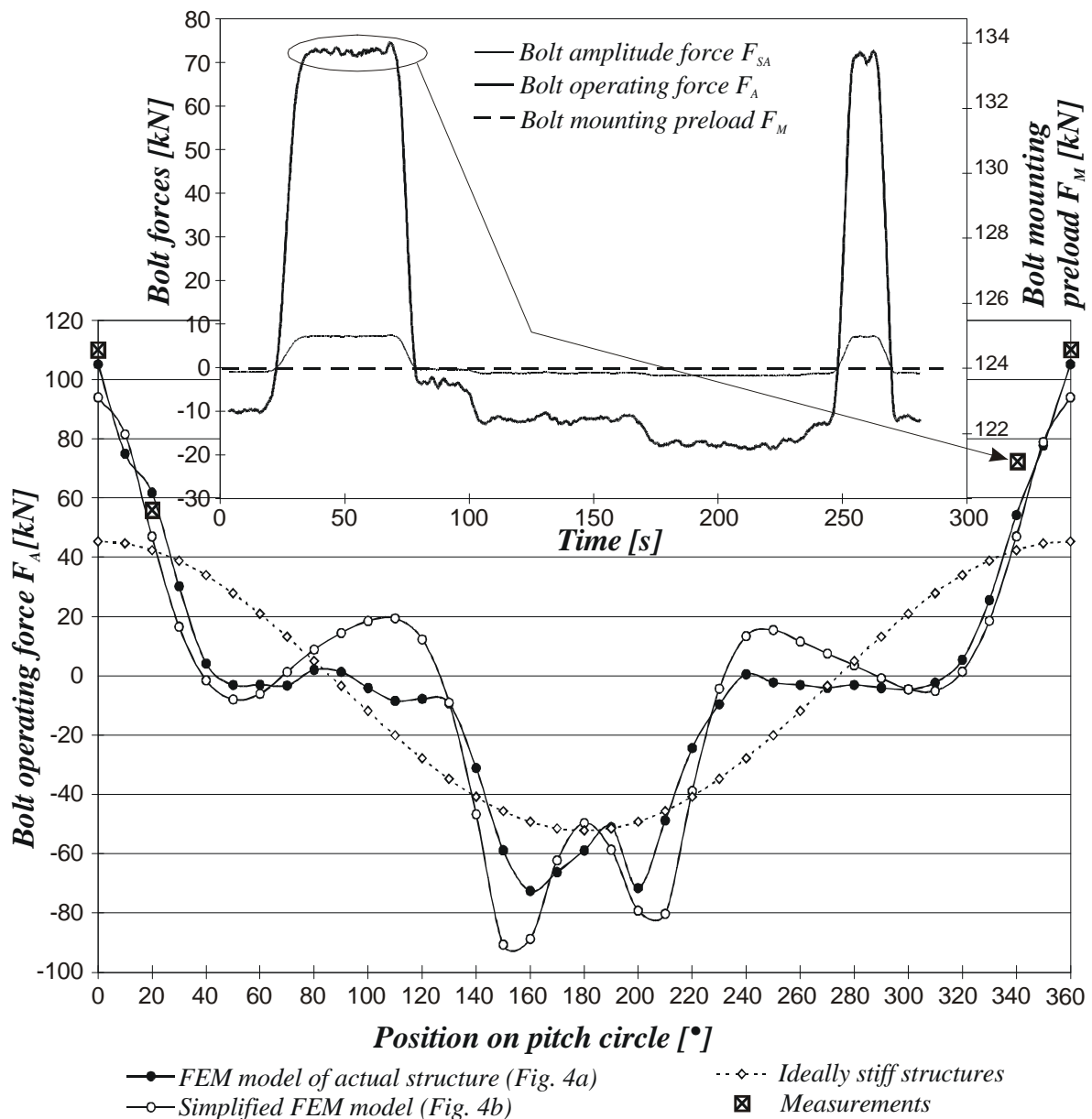
- The method can be further generalised by considering different stiffness information for each bolt and present it as numeric input value. Changing the data for bolt and flange stiffness is a relatively simple task.
- Structure elasticity matrices can be determined with a linear elastic FEM analysis which is quite straightforward and fast. Structures A and B are dealt with separately, which facilitates modelling and decreases computation times (Fig. 4).
- The method facilitates analyses of individual structure effects by allowing one of the structures to be considered an ideal stiff structure ( $[P]=a[I]$ ,  $a \ll 1$ ).
- If the external joint load changes, only a system of equations (11, 12) with a different load vector needs to be solved to perform the analysis.
- In case of minor changes in bolt positions or changes in the number of bolts distributed along the same (or almost the same) path, new elasticity matrices can be calculated with the 2D interpolation of the original elasticity matrix of the structure. It is advisable that in such cases the original elasticity matrix is defined with an FEM analysis for a sufficient number of nodes along the bolt distribution path. It is sensible to increase node density at positions where the elasticity of the structure changes faster and at positions where higher bolt concentration is expected in case of uneven bolt distribution.
- Often there is a need to analyse joints which allow for different mutual inter-orientation of structures A and B (rotation of structure B round the z-axis). In rotational connections, for example, bolts are used to fix rings of large-sized rolling bearings onto the structures. When applying the described method, changes in inter-orientation can be simulated with a corresponding movement of the elements of the rotational structure B elasticity matrix. The simulation of rotation is accurate for the angles determined by the angle between the x-axis of the co-ordinate system, and position vector to the k-th bolt.

The FEM analyses to determine elasticity matrices can nowadays be carried out on average workstations with appropriate CAE software in a relatively short time. Further analyses of bolted joints can be performed with appropriate software applications on an average personal computer.

#### 5. Comparison of calculated and measured values

The described method was first tested on a rotational connection in the mobile crane Metalna MTA 180. The aim of the test was to study the effect of uneven stiffness of the supporting structure of the crane (on a truck) and upper construction on which the hydraulic telescopic boom of the crane is fixed. Since our primary interest was the effect of uneven stiffness upon the forces in rolling elements, we

made relatively accurate FEM models of both structures by using shell elements (Fig. 4). For better comparison we later made highly simplified line FEM construction models by using beam elements. Elasticity matrices of both structures were calculated for 121 positions in the pitch circle ( $D_i = 1208$  mm) of evenly distributed rolling elements of a single row ball bearing. Pitch circles of bolted joints in the outer bearing ring with the supporting structure and the inner ring with the upper structure were not much different in diameter from the bolt pitch circle ( $D_o = 1265$  mm,  $D_i = 1150$  mm). In the bolt area structure stiffness did not undergo any significant changes. Consequently, the elasticity matrices of structures were calculated for 36 evenly distributed bolts from the original matrix with a 2D interpolation. In production of the bolted joint in the outer ring, elastic bolts M16x115 and a clamping length  $l_k = 75$  mm were used, while in the inner ring in the joint was executed with elastic bolts M16x85 and with a clamping length  $l_k = 50$  mm.



**Figure 5. Comparison of distribution of calculated and measured operating bolt loads between the bearing ring and the supporting structure of the mobile crane**

Fig. 5 shows the distribution of operating bolt forces applied to the outer bolt ring which are calculated with different methods and compared with the values measured in a life-size object. Seemingly strange external load with tilting moment  $T_y = 555,3$  kNm ( $T_x = 0$ ) and the axial force  $F_z = -123.3$  kN results from the inclination of the crane boom and the available weight on the test field.

Fig. 5 and other calculations and measurements show a substantial deviation of highest values for operating forces calculated with the described method (up to 135% higher) from the values deriving from the presumption on ideal stiff structures. Calculated value for the maximum operating force obtained with experience method recommended by large sized bearing producer Hoesch Rothe Erde in his catalogue [Hoesch Rothe Erde 1989] is only 15% higher than at stiff structures. Operating bolt loads calculated with described enhanced method are in good agreement with the measured values, which was further confirmed by results obtained in other load cases and in other inter-orientations of joined structures.

## 6. Conclusion

Compared with classic methods for calculation of operating bolt forces, which presume ideal stiff structures, the method presented in this paper requires a relatively high effort if better results are to be obtained. Nevertheless, the presented method which is based on the use of elasticity and stiffness matrices is rather generalised. The creation and use of appropriate software applications offers opportunities for automated matrix calculation. In comparison with the classic FEM analysis of complete joined structures, the complexity of FEM analyses would thus be significantly reduced along with the time needed to obtain appropriate results.

Judging from the experience, the presented method can be of great usability in certain typified bolted joints which require numerous variations in product development, especially as regards the form and size of bolts, the number of bolts and the distribution of bolts along a more or less limited path and which require calculations at different inter-orientations of joined structures.

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